

An Introduction to Probability Models for Marketing Research

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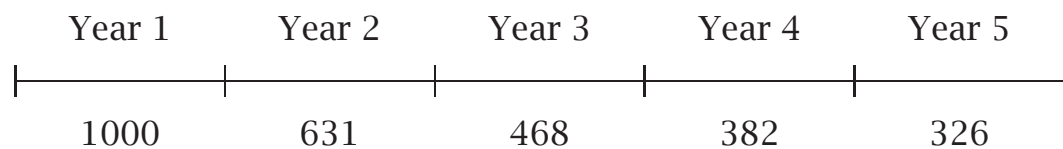
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Problem 1: Projecting Customer Retention Rates (Modelling Discrete-Time Duration Data)

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Motivating Problem

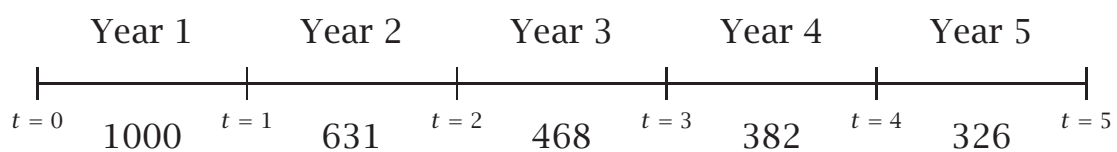
Consider a company with a subscription-based business model. 1000 customers are acquired at the beginning of Year 1 with the following pattern of renewals over the subsequent four years:



- How many customers will “survive” to Year 6, 7, ..., 13?
- What will the retention rates for this cohort look like for the next 8 years?

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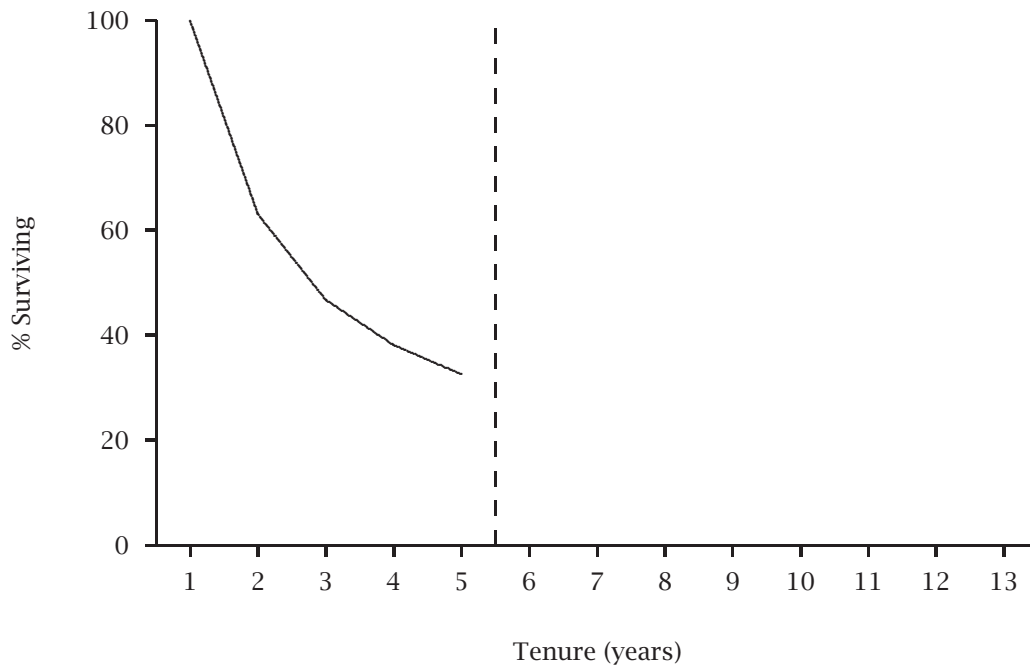
Notation and Terminology



- The empirical *survivor function* $S(t)$ is the proportion of the cohort that continue as a customer beyond t .
- Our modelling objective is to derive a mathematical function for $S(t)$, which can then be used to generate the desired forecasts.

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Modelling Objective



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Natural Starting Point

Project the survival curve using functions of time:

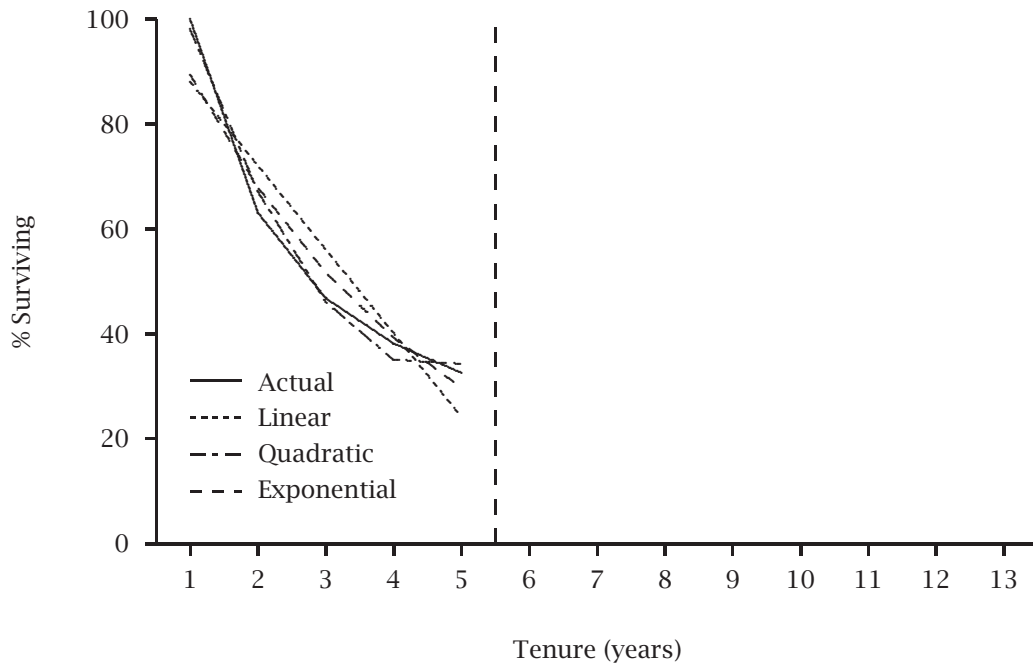
- Consider linear, quadratic, and exponential functions
- Let y = the proportion of customers surviving more than t years

$$y = 0.881 - 0.160t \quad R^2 = 0.868$$

$$y = 0.981 - 0.361t + 0.050t^2 \quad R^2 = 0.989$$

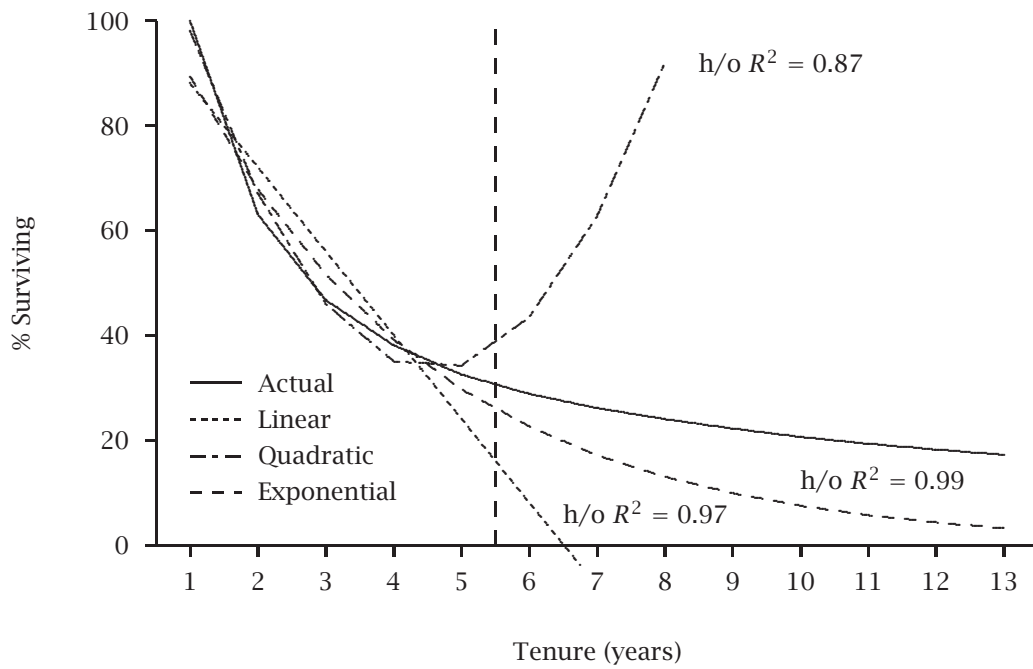
$$\ln(y) = -0.112 - 0.274t \quad R^2 = 0.954$$

Model Fit



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Survival Curve Projections



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Developing a Better Model (I)

At the end of each contract period, a customer makes the renewal decision by tossing a coin: \mathbb{H} \rightarrow renew, \mathbb{T} \rightarrow don't renew

Length of relationship

1 period	\mathbb{T}
2 periods	$\mathbb{H} \quad \mathbb{T}$
3 periods	$\mathbb{H} \quad \mathbb{H} \quad \mathbb{T}$
...	

$$P(t \text{ periods}) = \begin{cases} P(\mathbb{T}) & t = 1 \\ P(\mathbb{H}) \times P(t - 1 \text{ periods}) & t = 2, 3, \dots \end{cases}$$

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Developing a Better Model (I)

- i) $P(\mathbb{H}) = 1 - \theta$ is constant and unobserved.
- ii) All customers have the same “churn probability” θ .

	A	B	C	D	E
1	theta	0.2			
2					
3					
4	t	# Cust.	# Lost	P(die)	S(t)
5	0	1000			1.0000
6	1	631	=B1	0.2000	0.8000
7	2	468	163	0.1600	0.6400
8	3	382	86	=E5-D6	0.5120
9	4	=D6*(1-\$B\$1)		0.1024	0.4096
10					

Developing a Better Model (I)

More formally:

- Let the random variable T denote the duration of the customer's relationship with the firm.
- We assume that the random variable T has a geometric distribution with parameter θ :

$$\begin{aligned}P(T = t | \theta) &= \theta(1 - \theta)^{t-1}, \quad t = 1, 2, 3, \dots \\S(t | \theta) &= P(T > t | \theta) \\&= (1 - \theta)^t, \quad t = 0, 1, 2, 3, \dots\end{aligned}$$

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Estimating Model Parameters

Assuming

- i) the observed data were generated according to the “coin flipping” story of contract renewal, and
- ii) we know $P(\mathbb{T}) = \theta$,

the probability of the observed pattern of renewals is:

$$\begin{aligned}& [P(T = 1 | \theta)]^{369} [P(T = 2 | \theta)]^{163} [P(T = 3 | \theta)]^{86} \\& \quad \times [P(T = 4 | \theta)]^{56} [S(t | \theta)]^{326} \\& = [\theta]^{369} [\theta(1 - \theta)]^{163} [\theta(1 - \theta)^2]^{86} \\& \quad \times [\theta(1 - \theta)^3]^{56} [(1 - \theta)^4]^{326}\end{aligned}$$

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Estimating Model Parameters

- Suppose we have two candidate coins:

Coin A: $\theta = 0.2$

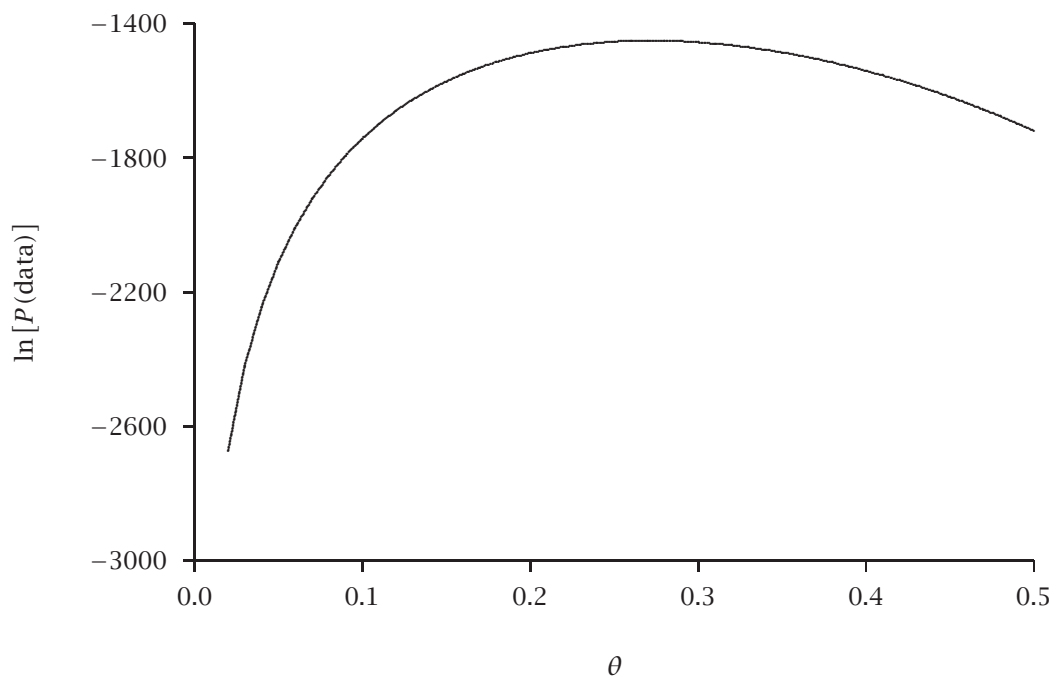
Coin B: $\theta = 0.5$

- Which coin is more likely to have generated the observed pattern of renewals across this set of 1000 customers?

θ	$P(\text{data} \theta)$	$\ln [P(\text{data} \theta)]$
0.2	6.00×10^{-647}	-1488.0
0.5	1.40×10^{-747}	-1719.7

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Estimating Model Parameters



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Estimating Model Parameters

We estimate the model parameters using the method of *maximum likelihood*:

- The likelihood function is defined as the probability of observing the data for a given set of the (unknown) model parameters.
- It is computed using the model and is viewed as a function of the model parameters:

$$L(\text{parameters} \mid \text{data}) = p(\text{data} \mid \text{parameters}).$$

- For a given dataset, the maximum likelihood estimates of the model parameters are those values that maximize $L(\cdot)$.
- It is typically more convenient to use the natural logarithm of the likelihood function — the log-likelihood function.

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Estimating Model Parameters

The log-likelihood function is given by:

$$\begin{aligned} LL(\theta \mid \text{data}) = & 369 \times \ln[P(T = 1 \mid \theta)] + \\ & 163 \times \ln[P(T = 2 \mid \theta)] + \\ & 86 \times \ln[P(T = 3 \mid \theta)] + \\ & 56 \times \ln[P(T = 4 \mid \theta)] + \\ & 326 \times \ln[S(4 \mid \theta)] \end{aligned}$$

The maximum value of the log-likelihood function is $LL = -1451.2$, which occurs at $\hat{\theta} = 0.272$.

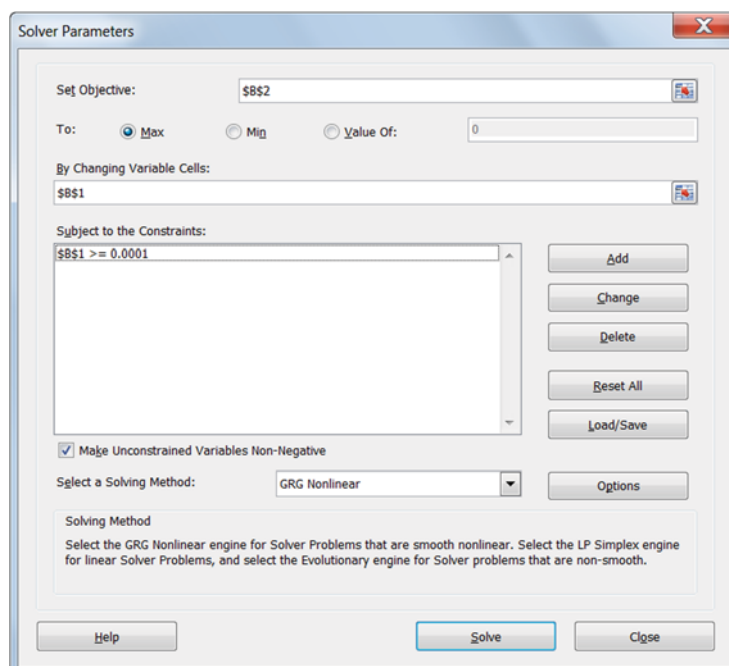
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Estimating Model Parameters

	A	B	C	D	E	F
1	theta	0.2				
2	LL	-1488.0	← =SUM(F6:F10)			
3						
4	t	# Cust.	# Lost	P(die)	S(t)	
5	0	1000			1.0000	
6	1	631	369	0.2000	0.8000	-593.88
7	2	468	163	0.1600	0.8400	-298.71
8	3	382	86	0.12	=C6*LN(D6)	
9	4	326	56	0.1024	0.4096	-127.62
10				=B9*LN(E9)		→
11						

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Estimating Model Parameters

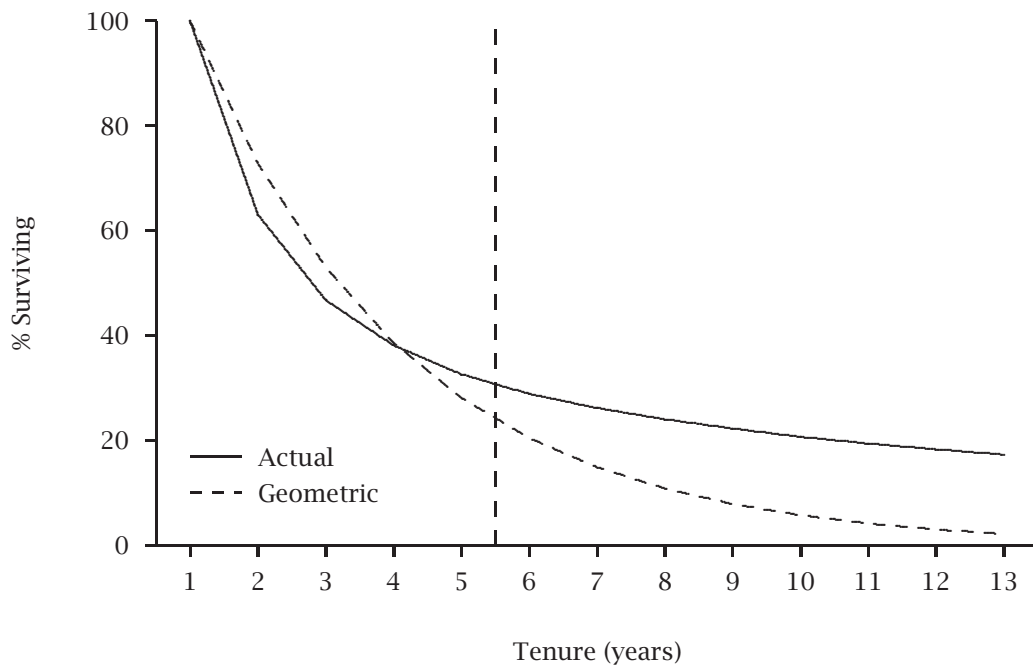


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	A	B	C	D	E	F
1	theta	0.272				
2	LL	-1451.2				
3						
4	t	# Cust.	# Lost	P(die)	S(t)	
5	0	1000			1.0000	
6	1	631	369	0.2717	0.7283	-480.88
7	2	468	163	0.1979	0.5305	-264.09
8	3	382	86	0.1441	0.3864	-166.60
9	4	326	56	0.1050	0.2814	-126.23
10	5			0.0764	0.2050	-413.36
11	6			0.0557	0.1493	
12	7			0.0406	0.1087	
13	8			0.0295	0.0792	
14	9			0.0215	0.0577	
15	10			0.0157	0.0420	
16	11			0.0114	0.0306	
17	12			0.0083	0.0223	

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Survival Curve Projection



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What's wrong with this story of customer contract-renewal behavior?

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Developing a Better Model (II)

Consider the following story of customer behavior:

- i) At the end of each period, an individual renews his contract with (constant and unobserved) probability $1 - \theta$.
- ii) “Churn probabilities” vary across customers.
 - Since we don't know any given customer's true value of θ , we treat it as a realization of a random variable (Θ).
 - We need to specify a probability distribution that captures how θ varies across customers (by giving us the probability of each possible value of θ).

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Developing a Better Model (II)

What is the probability that a randomly chosen new customer will cancel their contract at the end of period t ?

i) If we knew their θ , it would simply be

$$P(T = t | \theta) = \theta(1 - \theta)^{t-1}.$$

ii) Since we only know the distribution of Θ across the population, we compute

$$P(T = t) = E_{\Theta}[P(T = t | \theta)],$$

i.e., we evaluate $P(T = t | \theta)$ for each possible value of θ , weighting it by the probability of a randomly chosen new customer having that value of θ .

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Accounting for Heterogeneity (I)

• Suppose we have two (unobserved) segments:

$$\Theta = \begin{cases} \theta_1 & \text{with probability } \pi \\ \theta_2 & \text{with probability } 1 - \pi \end{cases}$$

• We compute $E_{\Theta}[P(T = t | \theta)]$:

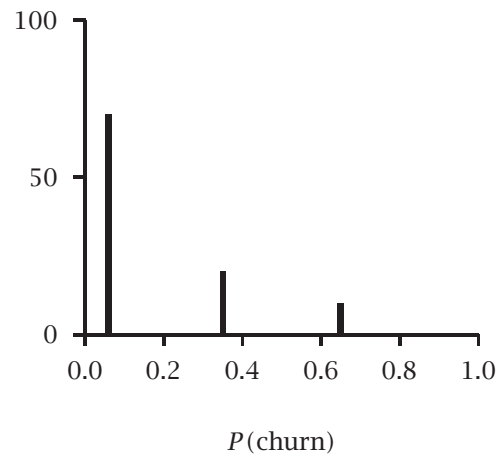
$$\begin{aligned} P(T = t | \theta_1, \theta_2, \pi) &= P(T = t | \Theta = \theta_1)P(\Theta = \theta_1) \\ &\quad + P(T = t | \Theta = \theta_2)P(\Theta = \theta_2) \\ &= \theta_1(1 - \theta_1)^{t-1}\pi + \theta_2(1 - \theta_2)^{t-1}(1 - \pi) \end{aligned}$$

• Likewise for three or four segments ...

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Vodafone Italia Churn Clusters

Cluster	$P(\text{churn})$	%CB
Low risk	0.06	70
Medium risk	0.35	20
High risk	0.65	10



Source: "Vodafone Achievement and Challenges in Italy" presentation (2003-09-12)

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Accounting for Heterogeneity (II)

- We move from a finite number of segments (a *finite mixture* model) to an infinite number of segments (a *continuous mixture* model).
- We choose a continuous distribution for Θ , with probability density function (pdf) $g(\theta \mid \text{parameters})$.
- We compute $E_{\Theta}[P(T = t \mid \theta)]$:

$$P(T = t \mid \text{parameters})$$

$$= \int_0^1 P(T = t \mid \Theta = \theta) g(\theta \mid \text{parameters}) d\theta .$$

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The Beta Distribution

- The beta distribution is a flexible (and mathematically convenient) two-parameter distribution bounded between 0 and 1:

$$g(\theta | \gamma, \delta) = \frac{\theta^{\gamma-1}(1-\theta)^{\delta-1}}{B(\gamma, \delta)},$$

where $\gamma, \delta > 0$ and $B(\gamma, \delta)$ is the beta function.

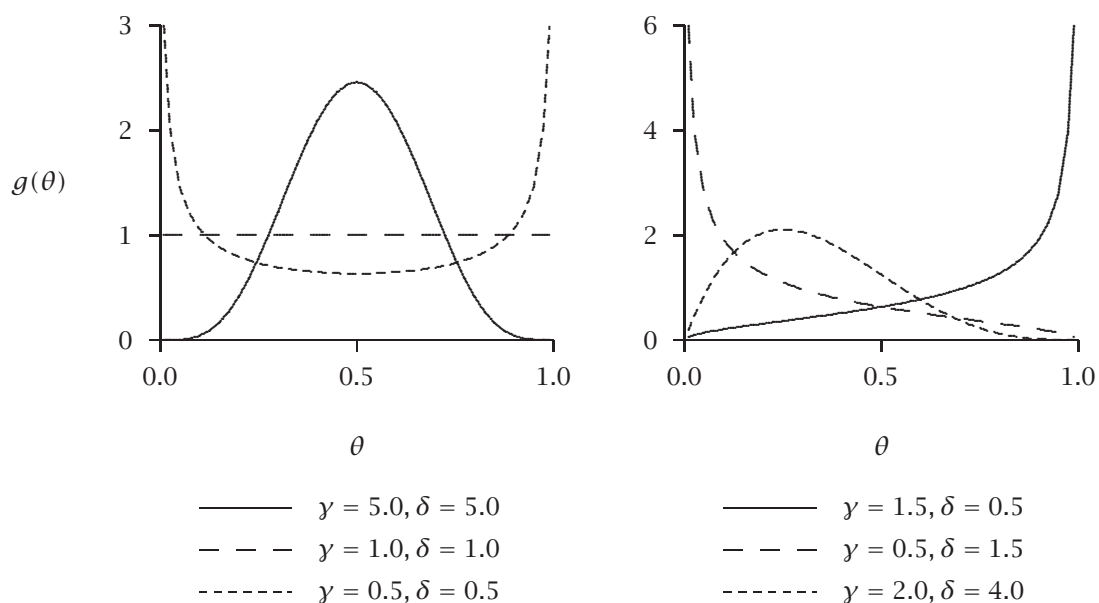
- The mean and variance are given by

$$E(\Theta) = \frac{\gamma}{\gamma + \delta}$$

$$\text{var}(\Theta) = \frac{\gamma\delta}{(\gamma + \delta)^2(\gamma + \delta + 1)}$$

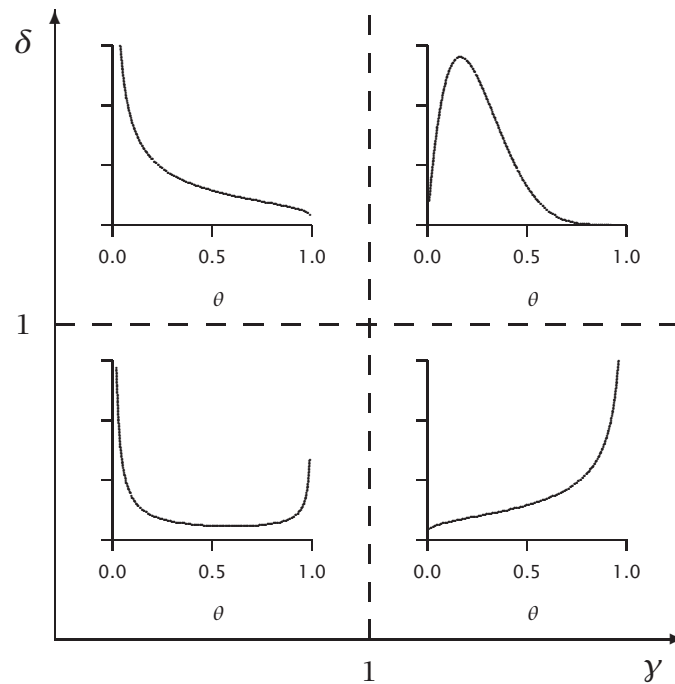
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Illustrative Beta Distributions



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Five General Shapes of the Beta Distribution



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The Beta Function

- The beta function $B(\gamma, \delta)$ is defined by the integral

$$B(\gamma, \delta) = \int_0^1 t^{\gamma-1} (1-t)^{\delta-1} dt, \quad \gamma > 0, \delta > 0,$$

and can be expressed in terms of gamma functions:

$$B(\gamma, \delta) = \frac{\Gamma(\gamma)\Gamma(\delta)}{\Gamma(\gamma + \delta)}.$$

- The gamma function $\Gamma(\gamma)$ is a generalized factorial, which has the recursive property $\Gamma(\gamma + 1) = \gamma\Gamma(\gamma)$. Since $\Gamma(0) = 1$, $\Gamma(n) = (n - 1)!$ for positive integer n .

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Developing a Better Model (II)

For a randomly chosen individual,

$$\begin{aligned}P(T = t | \gamma, \delta) &= \int_0^1 P(T = t | \theta) g(\theta | \gamma, \delta) d\theta \\&= \int_0^1 \theta(1 - \theta)^{t-1} \frac{\theta^{\gamma-1}(1 - \theta)^{\delta-1}}{B(\gamma, \delta)} d\theta \\&= \frac{1}{B(\gamma, \delta)} \int_0^1 \theta^\gamma (1 - \theta)^{\delta+t-2} d\theta \\&= \frac{B(\gamma + 1, \delta + t - 1)}{B(\gamma, \delta)}.\end{aligned}$$

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Developing a Better Model (II)

Similarly,

$$\begin{aligned}S(t | \gamma, \delta) &= \int_0^1 S(t | \theta) g(\theta | \gamma, \delta) d\theta \\&= \int_0^1 (1 - \theta)^t \frac{\theta^{\gamma-1}(1 - \theta)^{\delta-1}}{B(\gamma, \delta)} d\theta \\&= \frac{1}{B(\gamma, \delta)} \int_0^1 \theta^{\gamma-1}(1 - \theta)^{\delta+t-1} d\theta \\&= \frac{B(\gamma, \delta + t)}{B(\gamma, \delta)}.\end{aligned}$$

We call this *continuous mixture* model the beta-geometric (BG) distribution.

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Developing a Better Model (II)

We can compute BG probabilities using the following forward-recursion formula from $P(T = 1)$:

$$P(T = t) = \begin{cases} \frac{\gamma}{\gamma + \delta} & t = 1 \\ \frac{\delta + t - 2}{\gamma + \delta + t - 1} \times P(T = t - 1) & t = 2, 3, \dots \end{cases}$$

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Estimating Model Parameters

Assuming

- i) the observed data were generated according to the heterogeneous “coin flipping” story of contract renewal, and
- ii) we know γ and δ ,

the probability of the observed pattern of renewals is:

$$[P(T = 1 | \gamma, \delta)]^{369} [P(T = 2 | \gamma, \delta)]^{163} [P(T = 3 | \gamma, \delta)]^{86} \\ \times [P(T = 4 | \gamma, \delta)]^{56} [S(4 | \gamma, \delta)]^{326}$$

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Estimating Model Parameters

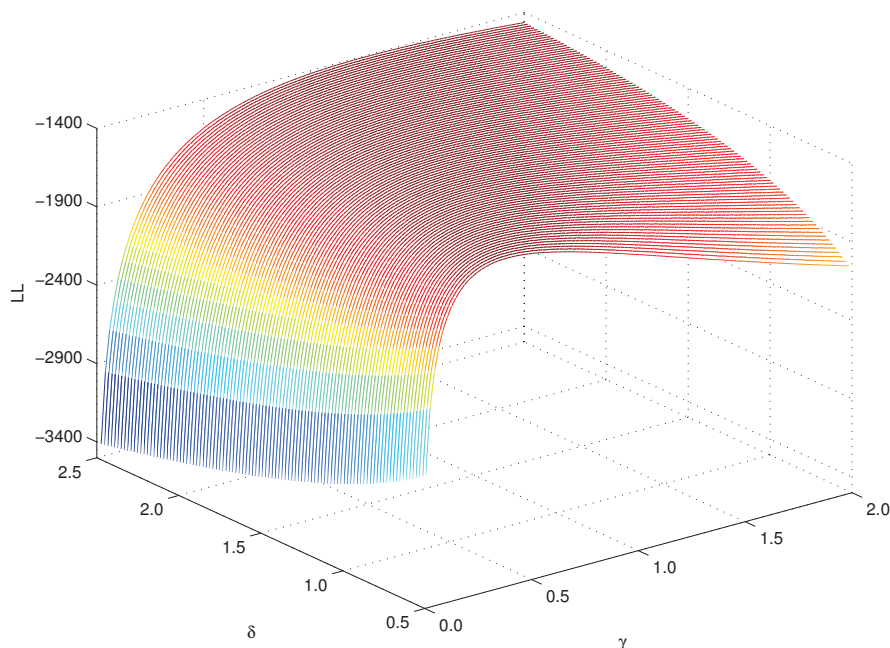
The log-likelihood function is given by:

$$\begin{aligned} LL(\gamma, \delta \mid \text{data}) = & 369 \times \ln[P(T = 1 \mid \gamma, \delta)] + \\ & 163 \times \ln[P(T = 2 \mid \gamma, \delta)] + \\ & 86 \times \ln[P(T = 3 \mid \gamma, \delta)] + \\ & 56 \times \ln[P(T = 4 \mid \gamma, \delta)] + \\ & 326 \times \ln[S(4 \mid \gamma, \delta)] \end{aligned}$$

The maximum value of the log-likelihood function is $LL = -1401.6$, which occurs at $\hat{\gamma} = 0.764$ and $\hat{\delta} = 1.296$.

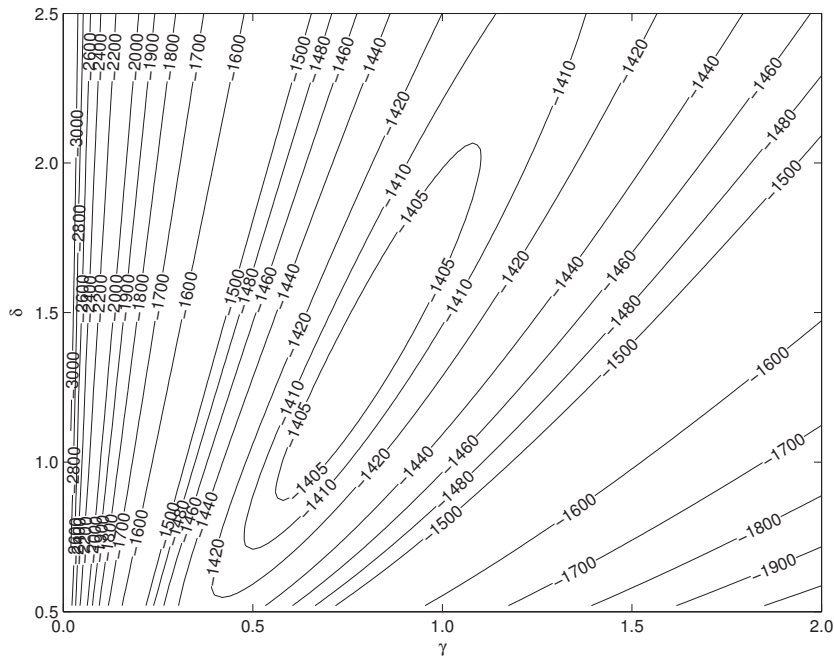
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Surface Plot of BG LL Function



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Contour Plot of BG LL Function



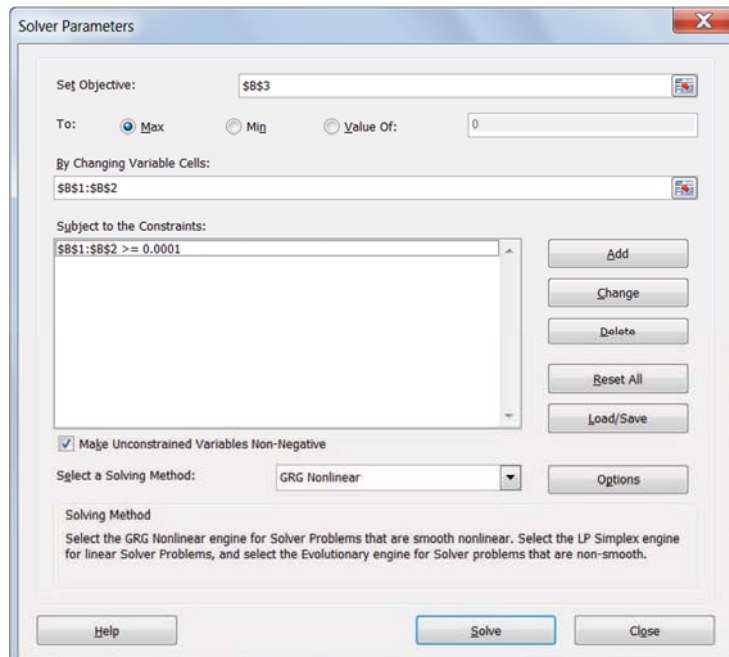
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Estimating Model Parameters

	A	B	C	D	E	F
1	gamma	1.000				
2	delta	1.000				
3	LL	-1454.0				
4						
5	t	# Cust.	# Lost	P(die)	S(t)	
6	0	1000			1.0000	
7	1	$=B1/(B1+B2)$	9	0.5000	0.5000	-255.77
8	2	468	163	0.1667	0.3333	-292.06
9	3	382	86	0.0833	0.2500	-213.70
10		$=D7*(\$B\$2+A8-2)/(\$B\$1+\$B\$2+A8-1)$.2000	-167.76
11						-524.68

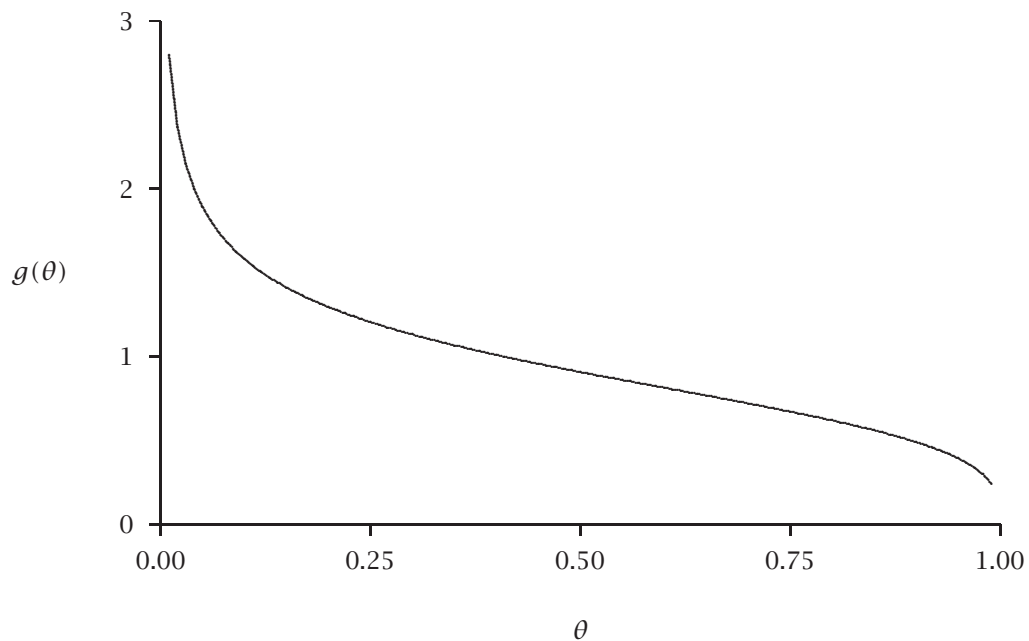
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Estimating Model Parameters



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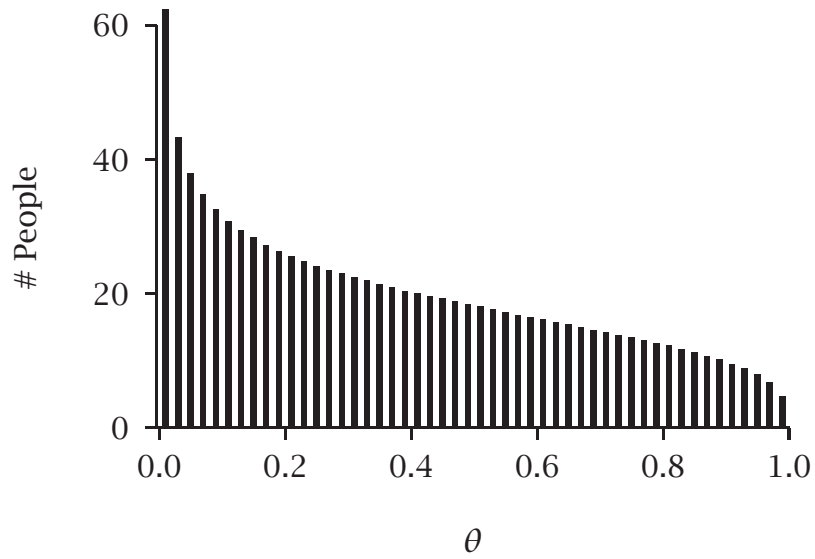
Estimated Distribution of Churn Probabilities



$$\hat{\gamma} = 0.764, \hat{\delta} = 1.296, \widehat{E(\Theta)} = 0.371$$

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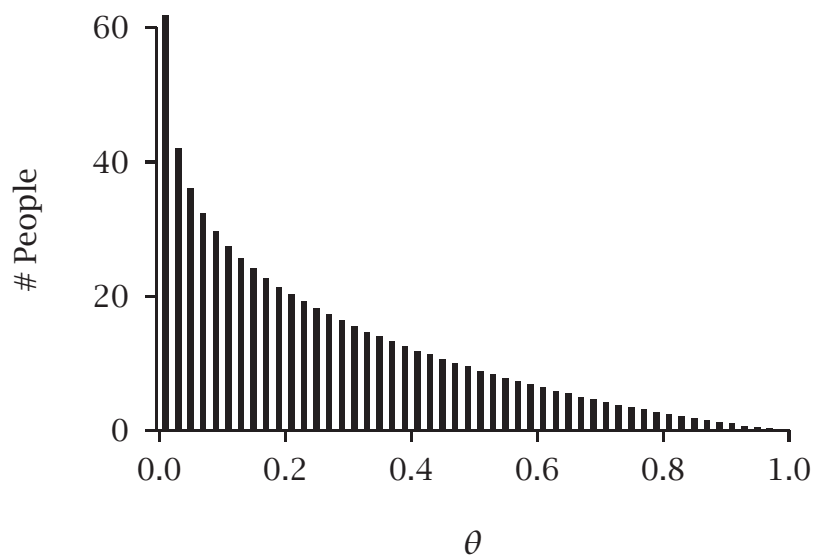
Year 1



$E(\Theta) = 0.371 \rightarrow$ expect $1000 \times (1 - 0.371) = 629$ customers to renew at the end of Year 1.

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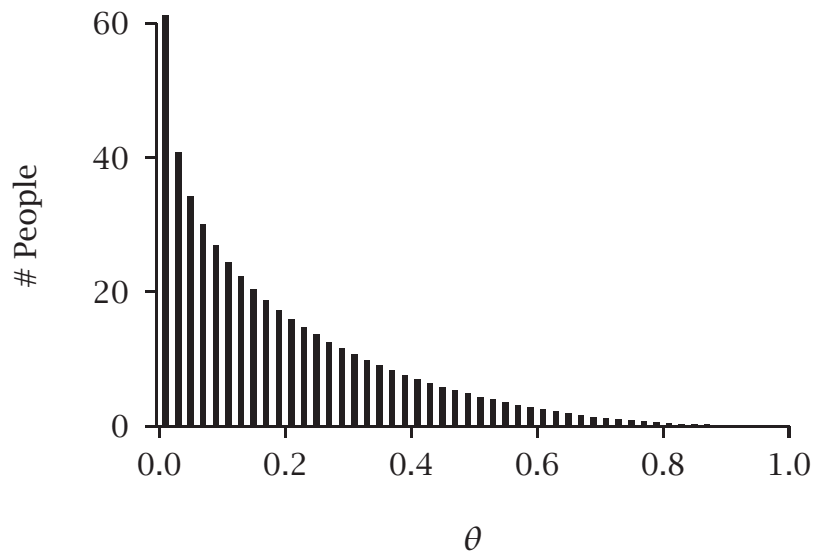
Year 2



$E(\Theta) = 0.250 \rightarrow$ expect $629 \times (1 - 0.250) = 472$ customers to renew at the end of Year 2.

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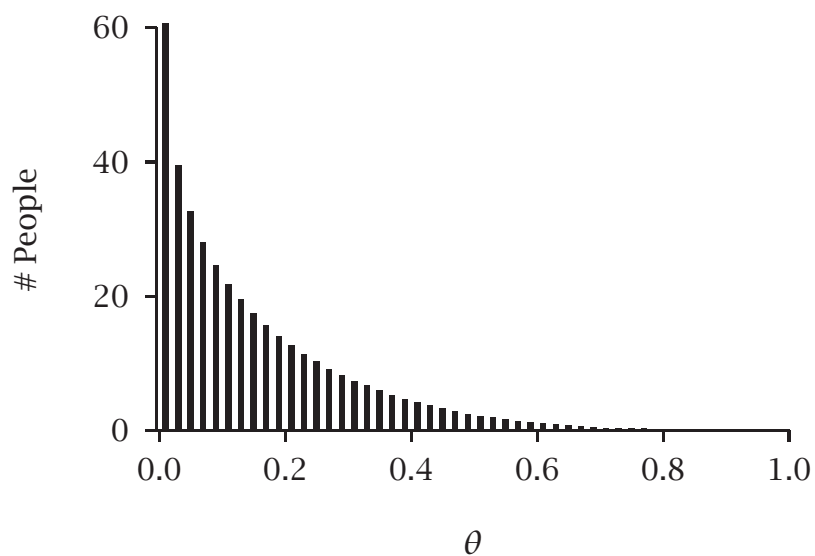
Year 3



$E(\Theta) = 0.188 \rightarrow$ expect $472 \times (1 - 0.188) = 383$ customers to renew at the end of Year 3.

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Year 4



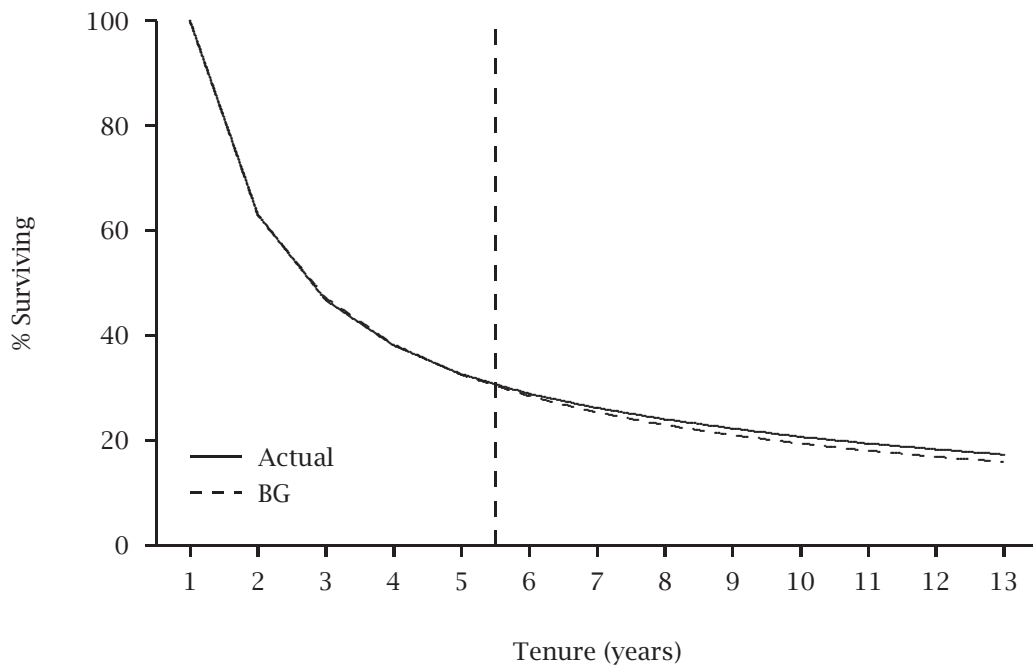
$E(\Theta) = 0.151 \rightarrow$ expect $383 \times (1 - 0.151) = 325$ customers to renew at the end of Year 4.

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	A	B	C	D	E	F
1	gamma	0.764				
2	delta	1.296				
3	LL	-1401.6				
4						
5	t	# Cust.	# Lost	P(die)	S(t)	
6	0	1000			1.0000	
7	1	631	369	0.3708	0.6292	-366.08
8	2	468	163	0.1571	0.4721	-301.74
9	3	382	86	0.0888	0.3833	-208.22
10	4	326	56	0.0579	0.3255	-159.59
11	5			0.0410	0.2845	-365.93
12	6			0.0308	0.2537	
13	7			0.0240	0.2296	
14	8			0.0194	0.2103	
15	9			0.0160	0.1943	
16	10			0.0134	0.1809	
17	11			0.0115	0.1694	
18	12			0.0099	0.1595	

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Survival Curve Projection



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Implied Retention Rates

- The retention rate for period t is defined as the proportion of customers who had renewed their contract at the end of period $t - 1$ who then renewed their contract at the end of period t .
- For any model of customer tenure with survivor function $S(t)$,

$$r_t = \frac{S(t)}{S(t-1)}.$$

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Implied Retention Rates

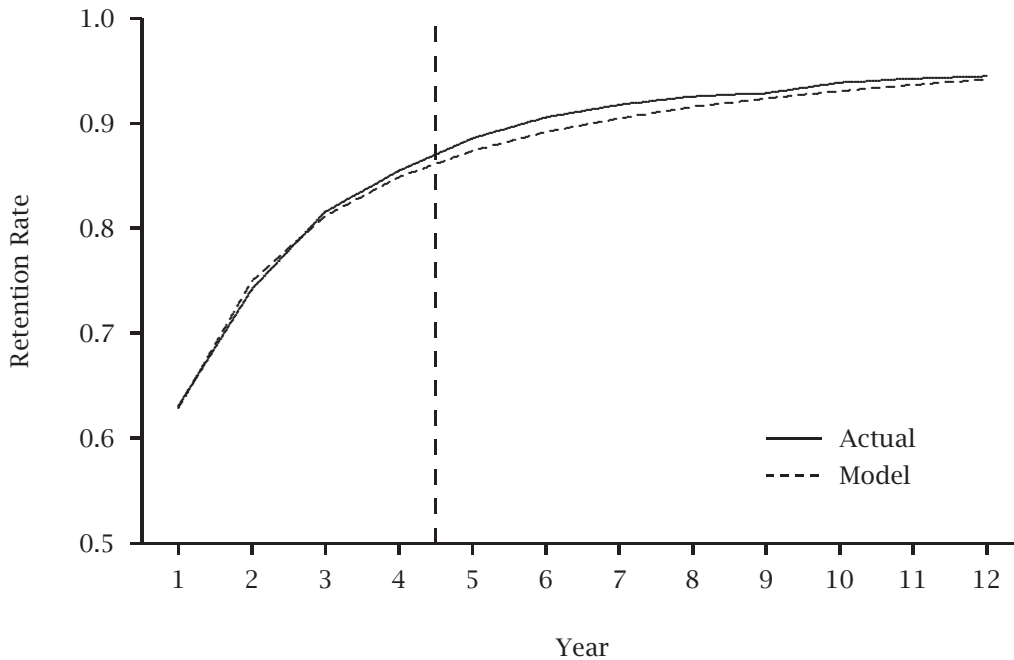
- For the BG model,

$$r_t = \frac{\delta + t - 1}{\gamma + \delta + t - 1}.$$

- An increasing function of time, even though the individual-level retention probability is constant.
- A sorting effect in a heterogeneous population.

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Projecting Retention Rates



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Concepts and Tools Introduced

- The concept of duration-time data, with a specific focus on single-event discrete-time data.
- The idea of building a “probability model” to characterize the observed behavior of interest.
- The method of maximum-likelihood as a means of estimating model parameters.
- The notion of finite- and continuous-mixture models.
- The beta-geometric (BG) distribution as a model of contract renewal behavior.
- Retention rate “dynamics.”

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Further Reading

Fader, Peter S. and Bruce G.S. Hardie (2007), "How to Project Customer Retention," *Journal of Interactive Marketing*, 21 (Winter), 76-90.

Fader, Peter S. and Bruce G.S. Hardie (2007), "How Not to Project Customer Retention." (<http://brucehardie.com/notes/016/>)

Potter, R. G. and M. P. Parker (1964), "Predicting the Time Required to Conceive," *Population Studies*, 18 (July), 99-116.

Lee, Ka Lok, Peter S. Fader, and Bruce G.S. Hardie (2007), "How to Project Patient Persistency," *FORESIGHT*, Issue 8, Fall, 31-35.

Buchanan, Bruce and Donald G. Morrison (1988), "A Stochastic Model of List Falloff with Implications for Repeat Mailings," *Journal of Direct Marketing*, 2 (Summer), 7-15.

An Introduction to Probability Models

“Winwood Reade is good upon the subject,” said Holmes. “He remarks that, while the individual man is an insoluble puzzle, in the aggregate he becomes a mathematical certainty. You can, for example, never foretell what any one man will do, but you can say with precision what an average number will be up to.”

Sir Arthur Conan Doyle, *The Sign of Four*.

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The Logic of Probability Models

- The actual data-generating process that lies behind any given data on buyer behavior embodies a huge number of factors.
 - Even if the actual process were completely deterministic, it would be impossible to measure all the variables that determine an individual’s buying behavior in any setting.
- ⇒ Any account of buyer behavior must be expressed in probabilistic/random/stochastic terms so as to account for our ignorance regarding (and/or lack of data on) all the determinants.

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The Logic of Probability Models

- Rather than try to tease out the effects of various marketing, personal, and situational variables, we embrace the notion of randomness and view the behavior of interest as the outcome of some probabilistic process.
- We propose a model of individual-level behavior that is “summed” across individuals (taking individual differences into account) to obtain a model of aggregate behavior.

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Applications of Probability Models

- Summarize and interpret patterns of market-level behavior
- Predict behavior in future periods, be it in the aggregate or at a more granular level (e.g., conditional on past behavior)
- Make inferences about behavior given summary measures
- Profile behavioral propensities of individuals
- Generate benchmarks/norms

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Building a Probability Model

- (i) Determine the marketing decision problem/
information needed.
- (ii) Identify the *observable* individual-level behavior of
interest.
 - We denote this by x .
- (iii) Select a probability distribution that characterizes this
individual-level behavior.
 - This is denoted by $f(x|\theta)$.
 - We view the parameters of this distribution as
individual-level *latent traits*.

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Building a Probability Model

- (iv) Specify a distribution to characterize the distribution
of the latent trait variable(s) across the population.
 - We denote this by $g(\theta)$.
 - This is often called the *mixing distribution*.
- (v) Derive the corresponding *aggregate* or *observed*
distribution for the behavior of interest:

$$f(x) = \int f(x|\theta)g(\theta) d\theta$$

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Building a Probability Model

- (vi) Estimate the parameters (of the mixing distribution) by fitting the aggregate distribution to the observed data.
- (vii) Use the model to solve the marketing decision problem/provide the required information.

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Outline

- Problem 1: Projecting Customer Retention Rates
(Modelling Discrete-Time Duration Data)
- Problem 2: Predicting New Product Trial
(Modelling Continuous-Time Duration Data)
- Problem 3: Estimating Concentration in Champagne Purchasing
(Modelling Count Data)
- Problem 4: Test/Roll Decisions in Segmentation-based Direct Marketing
(Modelling “Choice” Data)

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Problem 2:

Predicting New Product Trial

(Modelling Continuous-Time Duration Data)

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Setting

Ace Snackfoods, Inc. has developed a new shelf-stable juice product called Kiwi Bubbles. Before deciding whether or not to “go national” with the new product, the marketing manager for Kiwi Bubbles has decided to commission a year-long test market using IRI’s BehaviorScan service, with a view to getting a clearer picture of the product’s potential.

The product has now been under test in Grand Junction, CO for 24 weeks. The marketing manager for Kiwi Bubbles would like a forecast of the product’s year-end performance in the test market. First, she wants a forecast of the number of households that will have made a trial purchase by week 52.

On hand is a dataset documenting the number of households that have made a trial purchase by the end of each week. (The total size of the panel is 1499 households.)

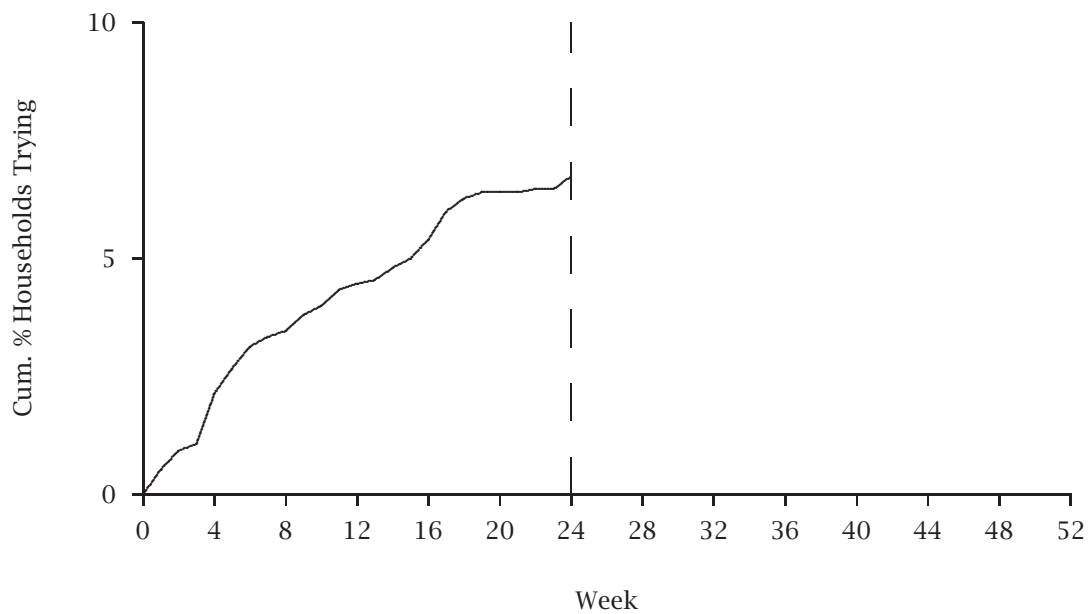
62

Kiwi Bubbles Cumulative Trial

Week	# Households	Week	# Households
1	8	13	68
2	14	14	72
3	16	15	75
4	32	16	81
5	40	17	90
6	47	18	94
7	50	19	96
8	52	20	96
9	57	21	96
10	60	22	97
11	65	23	97
12	67	24	101

63

Kiwi Bubbles Cumulative Trial



64

The Individual-Level Model

- Let the random variable T denote the time (since new product launch) at which the individual makes their first-ever (i.e., trial) purchase of the new product.
- We assume that T is distributed according the exponential distribution with parameter λ (which represents an individual's trial rate):

$$\begin{aligned}F(t | \lambda) &= P(T \leq t | \lambda) \\ &= 1 - e^{-\lambda t} \\ E(T | \lambda) &= \frac{1}{\lambda}\end{aligned}$$

65

Accounting for Heterogeneity

- Assume trial rates are distributed across the population according to a gamma distribution:

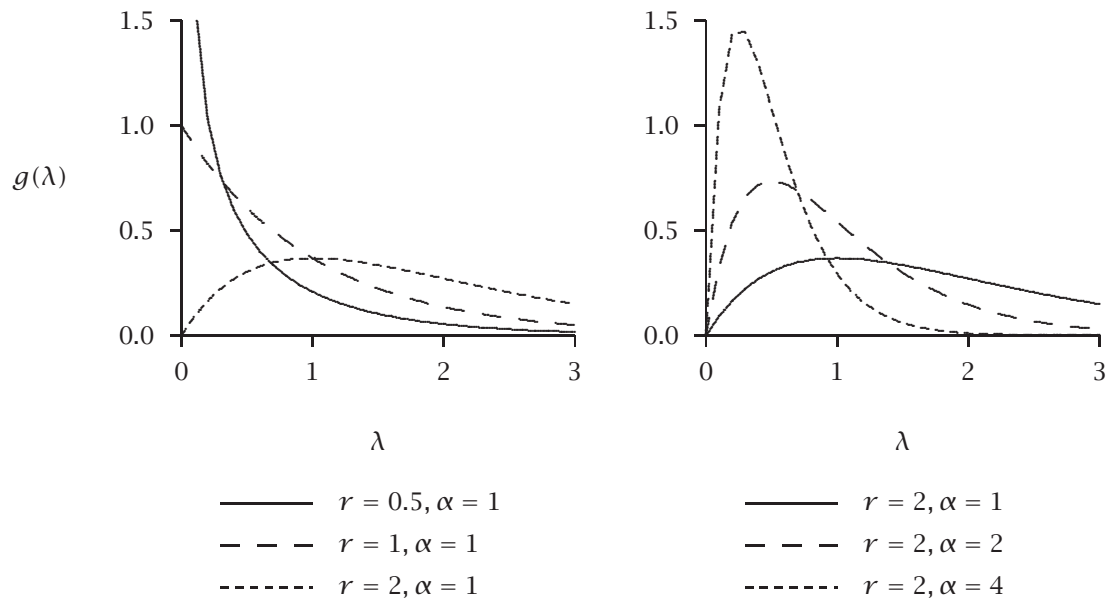
$$g(\lambda | r, \alpha) = \frac{\alpha^r \lambda^{r-1} e^{-\alpha\lambda}}{\Gamma(r)},$$

where r is the “shape” parameter and α is the “scale” parameter.

- The gamma distribution is a flexible (unimodal) distribution ... and is mathematically convenient.

66

Illustrative Gamma Density Functions



67

Accounting for Heterogeneity

- For a randomly chosen individual,

$$\begin{aligned}
 F(t | r, \alpha) &= \int_0^{\infty} F(t | \lambda) g(\lambda | r, \alpha) d\lambda \\
 &= \int_0^{\infty} (1 - e^{-\lambda t}) \frac{\alpha^r \lambda^{r-1} e^{-\alpha \lambda}}{\Gamma(r)} d\lambda \\
 &= 1 - \int_0^{\infty} \frac{\alpha^r \lambda^{r-1} e^{-\lambda(\alpha+t)}}{\Gamma(r)} d\lambda \\
 &= 1 - \left(\frac{\alpha}{\alpha + t} \right)^r .
 \end{aligned}$$

- This *gamma mixture of exponentials* is known as the Pareto distribution of the second kind (Pareto Type II) and the Lomax distribution.

68

Estimating Model Parameters

The log-likelihood function is defined as:

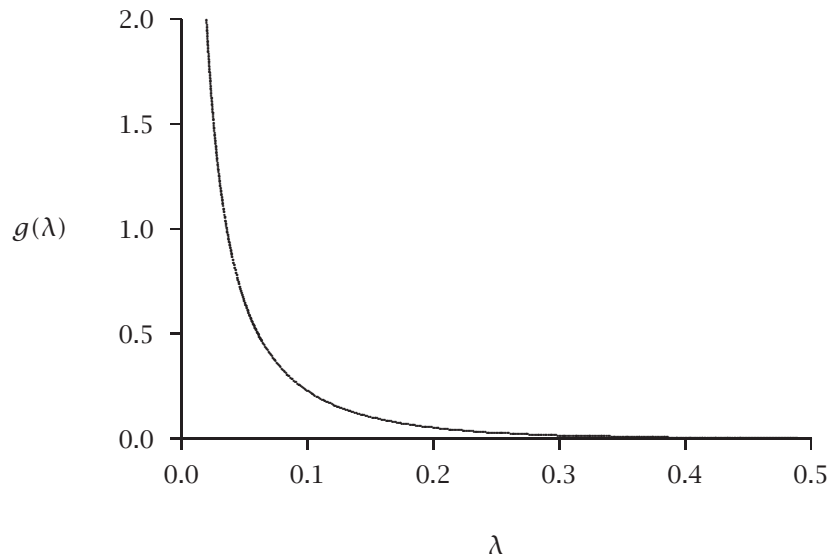
$$\begin{aligned}
 LL(r, \alpha | \text{data}) = & 8 \times \ln [F(1 | r, \alpha) - F(0 | r, \alpha)] + \\
 & 6 \times \ln [F(2 | r, \alpha) - F(1 | r, \alpha)] + \\
 & \dots + \\
 & 4 \times \ln [F(24 | r, \alpha) - F(23 | r, \alpha)] + \\
 & (1499 - 101) \times \ln[S(24 | r, \alpha)]
 \end{aligned}$$

The maximum value of the log-likelihood function is $LL = -681.4$, which occurs at $\hat{r} = 0.050$ and $\hat{\alpha} = 7.973$.

Estimating Model Parameters

	A	B	C	D	E	F
1	Product:	Kiwi Bubbles			r	1.000
2	Panelists:	1499			alpha	1.000
3			=SUM(F6:F30)	=>	LL	-4909.5
4		Cum_Trl				
5	Week	# HHs	Incr_Trl	F(t)	P(try week t)	
6		=1-(F\$2/(F\$2+A6))^F\$1	=>	0.5000	0.5000	-5.545
7	2	14	6	0.6667	0.1667	-10.751
8	3	16	2	0.7500	0.0833	-4.970
9	4	32	16	=D7-D6	0.0500	-47.932
10	5	40	8	0.8333	=C8*LN(E8)	-27.210
11	6	47	7	0.8571	0.0238	-26.164
12	7	50	3	0.8750	0.0179	-12.076
13	8	52	2	0.8889	0.0139	-8.553
14	9	57	5	0.9000	0.0111	-22.499
15	10	60	3	0.9091	0.0091	-14.101
29	24	101		0.9600	0.0017	-25.588
30				=(B2-B29)*LN(1-D29)	=>	-4499.988
31						

Estimated Distribution of Λ



$$\hat{r} = 0.050, \hat{\alpha} = 7.973$$

71

Forecasting Trial

- $F(t)$ represents the probability that a randomly chosen household has made a trial purchase by time t , where $t = 0$ corresponds to the launch of the new product.
- Let $T(t) =$ cumulative # households that have made a trial purchase by time t :

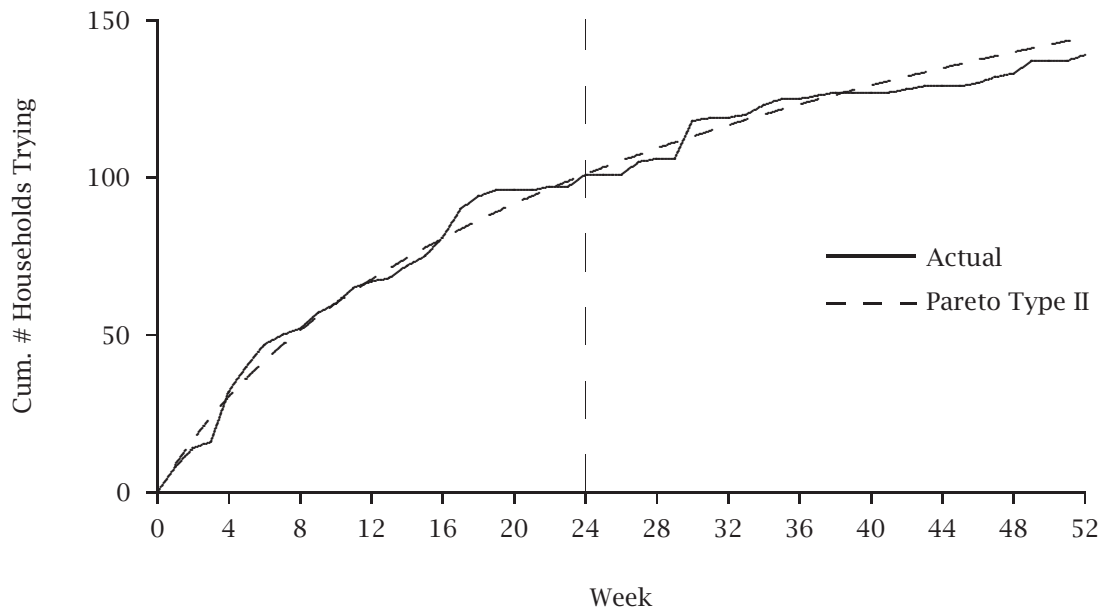
$$\begin{aligned} E[T(t)] &= N \times F(t) \\ &= N \left\{ 1 - \left(\frac{\hat{\alpha}}{\hat{\alpha} + t} \right)^{\hat{r}} \right\}, \end{aligned}$$

where N is the panel size.

- Use projection factors for market-level estimates.

72

Cumulative Trial Forecast



73

Concepts and Tools Introduced

- Modelling continuous-time (single-event) duration data
- Models of new product trial

74

Further Reading

Hardie, Bruce G. S., Peter S. Fader, and Michael Wisniewski (1998), “An Empirical Comparison of New Product Trial Forecasting Models,” *Journal of Forecasting*, **17** (June–July), 209–229.

Fader, Peter S., Bruce G. S. Hardie, and Robert Zeithammer (2003), “Forecasting New Product Trial in a Controlled Test Market Environment,” *Journal of Forecasting*, **22** (August), 391–410.

Morrison, Donald G. and David C. Schmittlein (1980), “Jobs, Strikes, and Wars: Probability Models for Duration,” *Organizational Behavior and Human Performance*, **25** (April), 224–251.

Meeker, William Q., and Luis A. Escobar (1998), *Statistical Methods for Reliability Data*, New York: John Wiley & Sons, Inc.

75

Forecasting New Product Sales

We have a model for cumulative trial sales up to t . What about repeat sales?

- Let $S(t)$ = total new product sales by time t :

$$S(t) = T(t) + R_1(t) + R_2(t) + \dots$$

where R_j = cumulative # households that have made a j th repeat purchase by time t .

- The “depth of repeat” approach of Eskin (1973) and Kalwani and Silk (1980) sees us modeling the time from trial to first repeat purchase, first repeat to second repeat purchase, etc. using the Pareto Type II model.

76

Forecasting New Product Sales

Eskin, Gerald J. (1973), "Dynamic Forecasts of New Product Demand Using a Depth of Repeat Model," *Journal of Marketing Research*, **10** (May), 115-129.

Fader, Peter S., Bruce G. S. Hardie, Robert Stevens, and Jim Findley (2003), "Forecasting New Product Sales in a Controlled Test Market Environment."

Kalwani, Manohar and Alvin J. Silk (1980), "Structure of Repeat Buying for New Packaged Goods," *Journal of Marketing Research*, **17** (August), 316-322.

Problem 3: Estimating Concentration in Champagne Purchasing

(Modelling Count Data)

Concentration 101

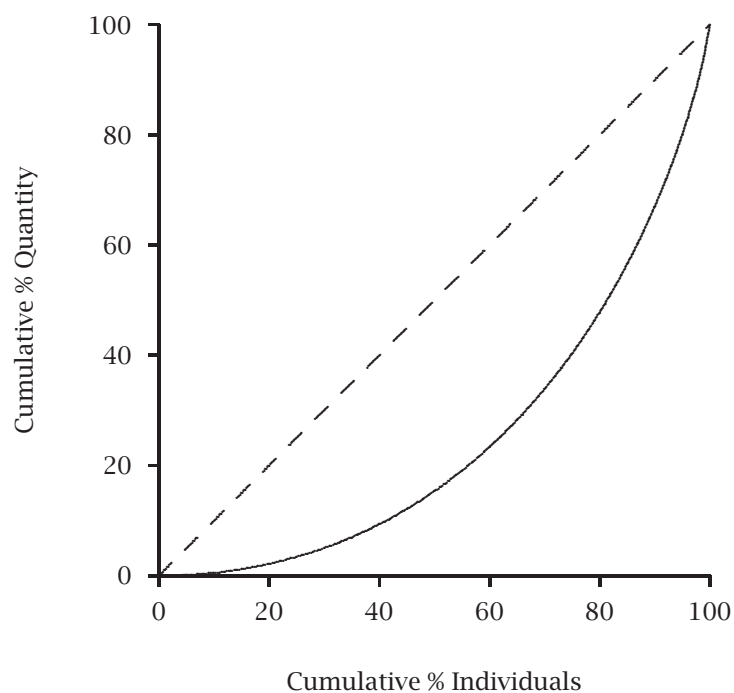
- Concentration in customer purchasing means that a small proportion of customers make a large proportion of the total purchases of the product (e.g., “80/20”).

higher concentration \Leftrightarrow greater inequality

- The *Lorenz curve* is used to illustrate the degree of inequality in the distribution of a quantity of interest (e.g., purchasing, income, wealth).
- The greater the curvature of the Lorenz Curve, the greater the concentration/inequality.

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Concentration 101



80

Concentration 101

- Every point on the Lorenz curve represents the $y\%$ of the quantity of interest accounted for by the bottom $x\%$ of all relevant individuals:

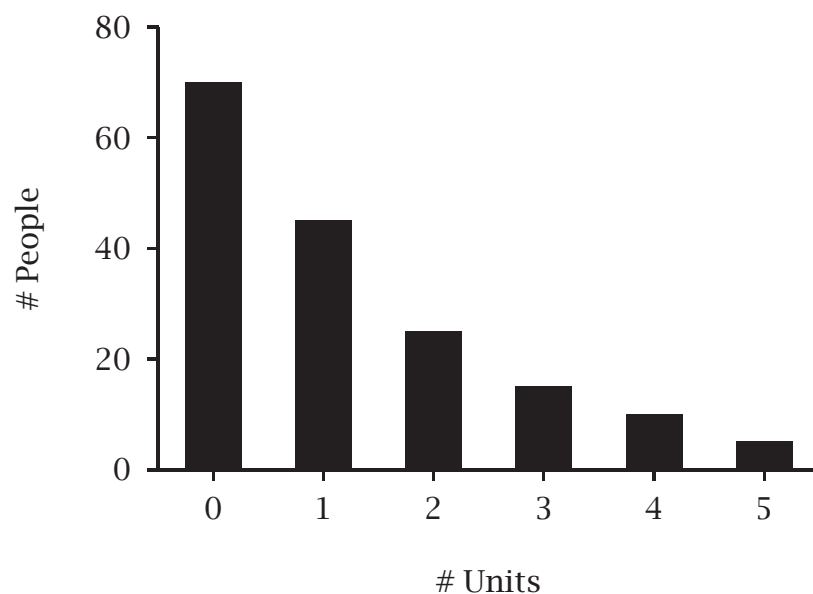
$$y = L(x)$$

- 80/20 represents a specific point on the Lorenz curve:
 $20 = L(80)$.
- The *Gini coefficient* is the ratio of the area between the 45° line (“line of perfect equality”) and the Lorenz curve to the area under the line of perfect equality.

81

Concentration 101

Hypothetical distribution of purchases ($n = 170$ people):



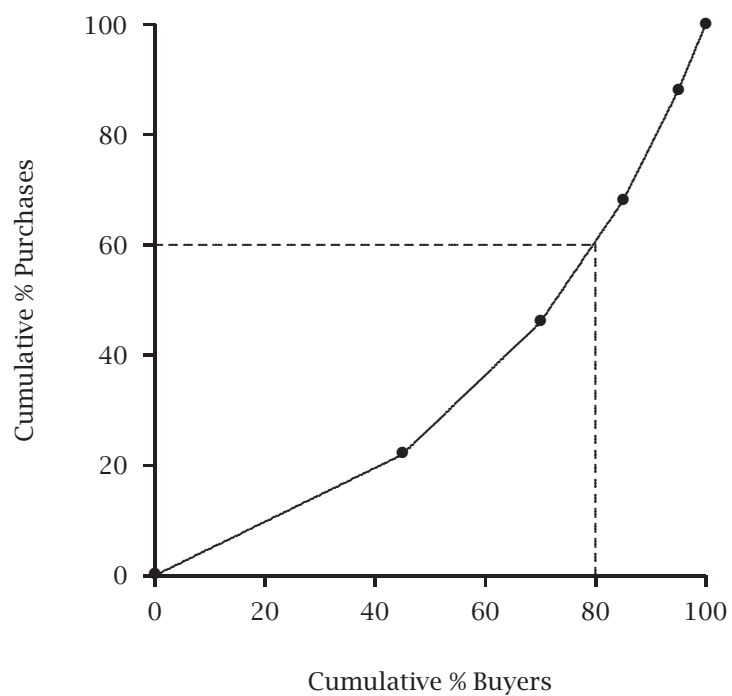
82

Concentration 101

# Units	# People	Total Units	% Buyers	% Purchases	Cum. % Buyers	Cum. % Purchases
0	70	0	0%	0%	0%	0%
1	45	45	45%	22%	45%	22%
2	25	50	25%	24%	70%	46%
3	15	45	15%	22%	85%	68%
4	10	40	10%	20%	95%	88%
5	5	25	5%	12%	100%	100%

83

Lorenz Curve



84

Calculations Revisited

	A	B	C	D	E	F	G	H	I	J
1	x	f_x	% buyers			P(X=x)	% buyers			
2	0	70			=B2/\$B\$8 -->	0.412				
3	1	45	45%	<-- =B3/(\$B\$8-\$B\$2)		0.265	45%	<-- =F3/(1-\$F\$2)		
4	2	25	25%			0.147	25%			
5	3	15	15%			0.088	15%			
6	4	10	10%			0.059	10%			
7	5	5	5%			0.029	5%			
8		170								
9										
10	x	f_x	Tot units	% purch.		P(X=x)	x P(X=x)	% purch.		
11	0	70	0	0%		0.412	0.000	0%		
12	1	45	45	22%	<-- =C12/\$C\$17	0.265	0.265	22%	<-- =G12/\$G\$17	
13	2	25	50	24%		0.147	0.294	24%		
14	3	15	45	22%		0.088	0.265	22%		
15	4	10	40	20%		0.059	0.235	20%		
16	5	5	25	12%		0.029	0.147	12%		
17		170	205		=SUM(G11:G16) -->		1.206			
18	average		1.206	<-- =C17/B17						

85

Problem

Consider the following data on the number of bottles of champagne purchased in a year by a sample of 568 French households:

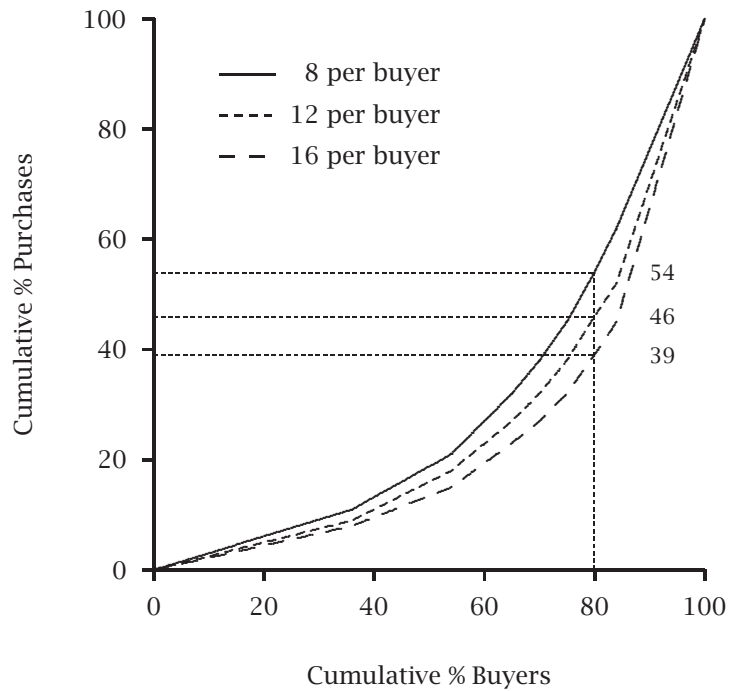
# Bottles	0	1	2	3	4	5	6	7	8+
Frequency	400	60	30	20	8	8	9	6	27

What percentage of buyers account for 80% of champagne purchasing? 50% of champagne purchasing?

Data source: Gourieroux and Visser (*Journal of Econometrics*, 1997)

86

Associated Lorenz Curves



87

Modelling Objective

We need to infer the full distribution from the right-censored data ... from which we can create the Lorenz curve.

- Develop a model that enables us to estimate the number of people purchasing 0, 1, 2, ..., 7, 8, 9, ... bottles of champagne in a year.

Model Development

- Let the random variable X denote the number of bottles purchased in a year.
- At the individual-level, X is assumed to be Poisson distributed with (purchase) rate parameter λ :

$$P(X = x | \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- Purchase rates (λ) are distributed across the population according to a gamma distribution:

$$g(\lambda | r, \alpha) = \frac{\alpha^r \lambda^{r-1} e^{-\alpha\lambda}}{\Gamma(r)}$$

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Model Development

- For a randomly chosen individual,

$$\begin{aligned} P(X = x | r, \alpha) &= \int_0^{\infty} P(X = x | \lambda) g(\lambda | r, \alpha) d\lambda \\ &= \frac{\Gamma(r+x)}{\Gamma(r)x!} \left(\frac{\alpha}{\alpha+1}\right)^r \left(\frac{1}{\alpha+1}\right)^x \end{aligned}$$

- This *gamma mixture of Poissons* is called the Negative Binomial Distribution (NBD).
- The mean and variance of the NBD are

$$E(X | r, \alpha) = \frac{r}{\alpha} \text{ and } \text{var}(X | r, \alpha) = \frac{r}{\alpha} + \frac{r}{\alpha^2}.$$

90

Computing NBD Probabilities

- Note that

$$\frac{P(X = x)}{P(X = x - 1)} = \frac{r + x - 1}{x(\alpha + 1)}$$

- We can therefore compute NBD probabilities using the following *forward recursion* formula:

$$P(X = x | r, \alpha) = \begin{cases} \left(\frac{\alpha}{\alpha + 1}\right)^r & x = 0 \\ \frac{r + x - 1}{x(\alpha + 1)} \times P(X = x - 1) & x \geq 1 \end{cases}$$

91

Estimating Model Parameters

The log-likelihood function is defined as:

$$\begin{aligned} LL(r, \alpha | \text{data}) = & 400 \times \ln[P(X = 0)] + \\ & 60 \times \ln[P(X = 1)] + \\ & \dots + \\ & 6 \times \ln[P(X = 7)] + \\ & 27 \times \ln[P(X \geq 8)] \end{aligned}$$

The maximum value of the log-likelihood function is $LL = -646.96$, which occurs at $\hat{r} = 0.161$ and $\hat{\alpha} = 0.129$.

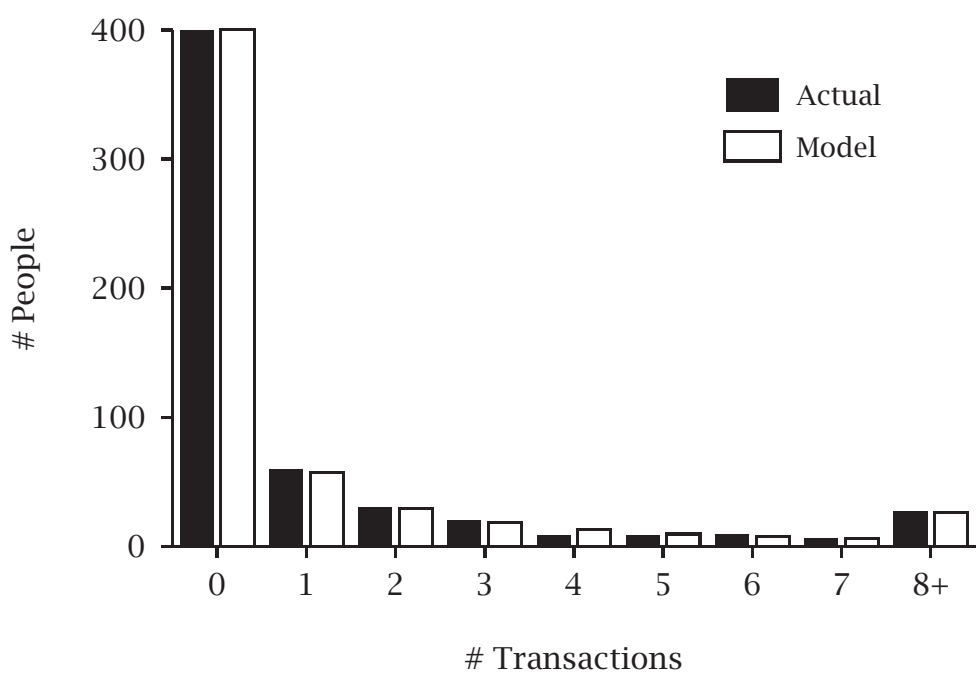
92

Estimating Model Parameters

	A	B	C	D
1	r	0.161		
2	alpha	0.129		
3	LL	-646.96	=LN(C6)*B6	
4				
5	x	f_x	P(X=x)	LL
6	0	400	0.7052	-139.72
7	1	60	0.1006	-137.80
8	2	30	0.0517	-88.86
9			0.0330	-68.23
10	4	8	0.0231	-30.14
11	5	8	0.0170	-32.59
12	6	6	0.0106	-39.11
13				27.57
14	8+	27	0.0463	-82.96
15		568		
16				
17			=1-SUM(C6:C13)	
18				

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Model Fit



94

Chi-square Goodness-of-Fit Statistic

Does the distribution $F(x|\boldsymbol{\theta})$, with s model parameters denoted by $\boldsymbol{\theta}$, provide a good fit to the sample data?

- Divide the sample into k mutually exclusive and collectively exhaustive groups.
- Let f_i ($i = 1, \dots, k$) be the number of sample observations in group i , p_i the probability of belonging to group i , and n the sample size.

95

Chi-square Goodness-of-Fit Statistic

- Compute the test statistic

$$\chi^2 = \sum_{i=1}^k \frac{(f_i - np_i)^2}{np_i}$$

- Reject the null hypothesis that the observed data come from $F(x|\boldsymbol{\theta})$ if the test statistic is greater than the critical value (i.e., $\chi^2 > \chi_{.05, k-s-1}^2$).
- The critical value can be computed in Excel 2010 using the CHISQ.INV.RT function (and the corresponding p-value using the CHISQ.DIST.RT function).

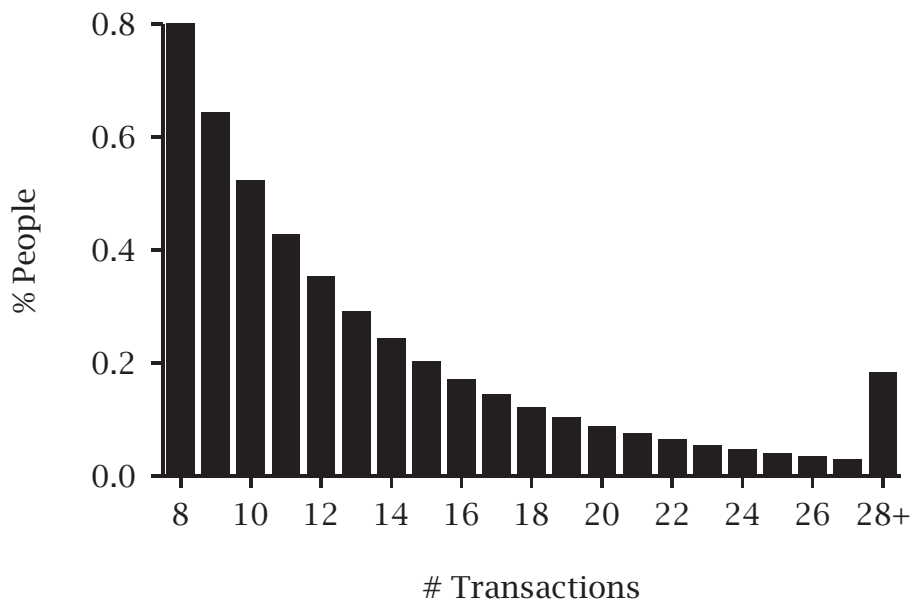
96

Model Fit

	A	B	C	D	E	F
1	r	0.161				
2	alpha	0.129				
3	LL	-646.96			=B\$15*C6	
4						
5	x	f_x	P(X=x)	LL	E(f_x)	(O-E)^2/E
6	0	400	0.7052	-139.72	400.5	0.001
7	1	60	0.1006	-137.80	57.1	0.144
8	2	30	0.0517	-88.86	29.4	0.013
9	3	20	0.0330	-68.23	18.7	0.084
10	4	8	0.0231	-30.14	13.1	1.997
11	5	8	0.0170	-3	= (B9-E9)^2/E9	0.288
12	6	9	0.0130	-39.11	7.4	0.362
13	7	6	0.0101	-27.57	5.7	0.012
14	8+	27	0.0463	-82.96	26.3	0.019
15		568				2.919
16						
17					df	6
18					Chi-sq crit	12.592
19					p-value	0.819

97

Decomposing the 8+ Cell



The mean for this group of people is 13.36 purchases per buyer ... but with great variability.

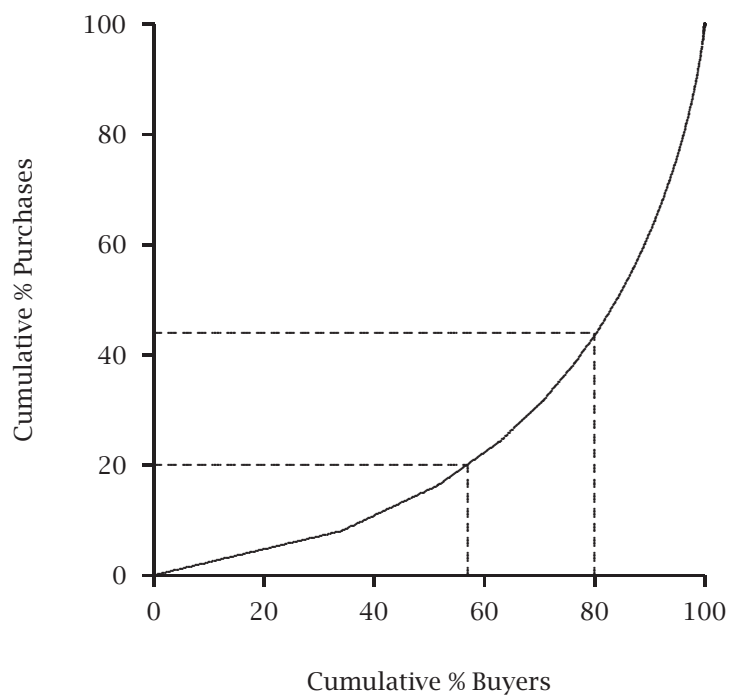
98

Creating the Lorenz Curve

	A	B	C	D	E	F
1	r	0.161	E(X)	1.248		
2	alpha	0.129				
3					Cumulative	
4	x	P(X=x)	% Cust.	% Purch.	% Cust.	% Purch.
5	0	0.7052			0	0
6	1	0.1006	0.3412	0.0806	0.3412	0.0806
7	2	0.0517	0.1754	0.0829	0.5166	0.1635
8		$=B6/(1-B5)$	0.1119	0.0793	0.6286	0.2429
9		0.0231	0.0783	0.0740	0.7069	0.3169
10	5	0.01	$=A6*B6/5D51$	0.0682	0.7646	0.3851
11	6	0.0130	0.0440	0.0624	0.8086	0.4475
12	7	0.0101	0.0343	0.0567	0.8429	0.5042
104	99	0.0000	5.29E-08	1.24E-06	1.0000	1.0000
105	100	0.0000	4.64E-08	1.10E-06	1.0000	1.0000

99

Lorenz Curve for Champagne Purchasing



100

Concepts and Tools Introduced

- The concept of count data.
- The NBD as a model for count data.
- The notion of concentration, and the Lorenz curve as a means of illustrating the level of “inequality” in the quantity of interest.

101

Further Reading

Ehrenberg, A. S. C. (1959), “The Pattern of Consumer Purchases,” *Applied Statistics*, **8** (March), 26–41.

Greenwood, Major and G. Udny Yule (1920), “An Inquiry into the Nature of Frequency Distributions Representative of Multiple Happenings with Particular Reference to the Occurrence of Multiple Attacks of Disease or of Repeated Accidents,” *Journal of the Royal Statistical Society*, **83** (March), 255–279.

Greene, Jerome D. (1982), *Consumer Behavior Models for Non-Statisticians*, New York: Praeger.

102

Further Reading

Ehrenberg, A. S. C. (1988), *Repeat-Buying*, 2nd edn, London: Charles Griffin & Company, Ltd. (Available at <http://www.empgens.com/ArticlesHome/Volume5/RepeatBuying.html>)

Morrison, Donald G. and David C. Schmittlein (1988), "Generalizing the NBD Model for Customer Purchases: What Are the Implications and Is It Worth the Effort?" *Journal of Business and Economic Statistics*, **6** (April), 145-159.

Schmittlein, David C., Lee G. Cooper, and Donald G. Morrison (1993), "Truth in Concentration in the Land of (80/20) Laws," *Marketing Science*, **12** (Spring), 167-183.

Problem 4: **Test/Roll Decisions in** **Segmentation-based Direct Marketing** (Modelling "Choice" Data)

The “Segmentation” Approach

- i) Divide the customer list into a set of (homogeneous) segments.
- ii) Test customer response by mailing to a random sample of each segment.
- iii) Rollout to segments with a response rate (RR) above some cut-off point,

$$\text{e.g., } RR > \frac{\text{cost of each mailing}}{\text{unit margin}}$$

105

Ben’s Knick Knacks, Inc.

- A consumer durable product (unit margin = \$161.50, mailing cost per 10,000 = \$3343)
- 126 segments formed from customer database on the basis of past purchase history information
- Test mailing to 3.24% of database

106

Ben's Knick Knacks, Inc.

Standard approach:

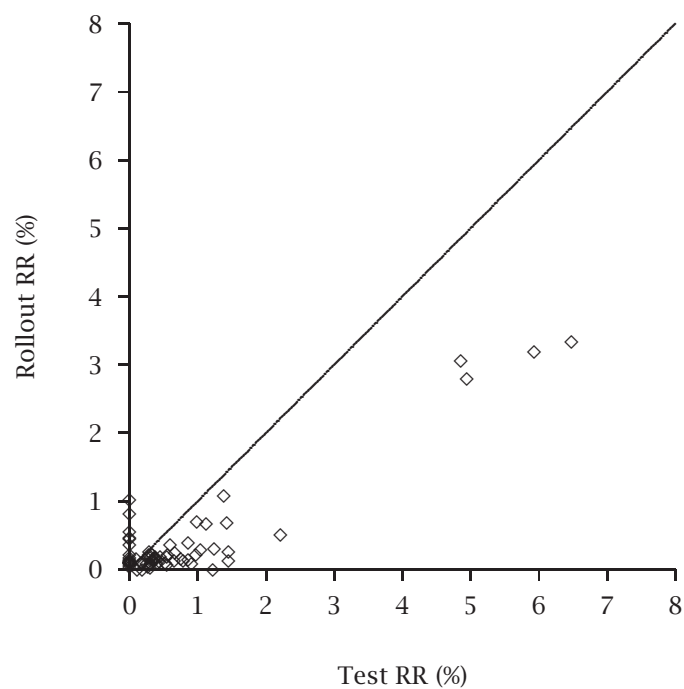
- Rollout to all segments with

$$\text{Test RR} > \frac{3,343/10,000}{161.50} = 0.00207$$

- 51 segments pass this hurdle

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Test vs. Actual Response Rate



108

Modelling Objective

Develop a model to help the manager estimate each segment's "true" response rate given the (limited) test data.

109

Model Development

Notation

N_s = size of segment s ($s = 1, \dots, S$)

m_s = # members of segment s tested

X_s = # responses to test in segment s

Assumptions

- i) All members of segment s have the same (unknown) response probability $\theta_s \Rightarrow X_s$ is a binomial random variable:

$$P(X_s = x_s | m_s, \theta_s) = \binom{m_s}{x_s} \theta_s^{x_s} (1 - \theta_s)^{m_s - x_s}$$

110

Distribution of Response Probabilities

ii) Heterogeneity in θ_s is captured by a beta distribution:

$$g(\theta_s | \alpha, \beta) = \frac{\theta_s^{\alpha-1} (1 - \theta_s)^{\beta-1}}{B(\alpha, \beta)}$$

It follows that the aggregate distribution of responses to a mailing of size m_s is given by

$$\begin{aligned} P(X_s = x_s | m_s, \alpha, \beta) &= \int_0^1 P(X_s = x_s | m_s, \theta_s) g(\theta_s | \alpha, \beta) d\theta_s \\ &= \binom{m_s}{x_s} \frac{B(\alpha + x_s, \beta + m_s - x_s)}{B(\alpha, \beta)}. \end{aligned}$$

This is known as the beta-binomial (BB) distribution.

111

Numerical Evaluation of the Beta Function

- Not all computing environments have a beta function.
- Recall

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}$$

- We typically have a function that evaluates $\ln(\Gamma(\cdot))$.
- In Excel we have `gamma ln`:

$$\Gamma(\alpha) = \exp(\text{gamma ln}(\alpha))$$

$$B(\alpha, \beta) = \exp(\text{gamma ln}(\alpha) + \text{gamma ln}(\beta) - \text{gamma ln}(\alpha + \beta))$$

112

Estimating Model Parameters

The log-likelihood function is defined as:

$$LL(\alpha, \beta | \text{data})$$

$$\begin{aligned}
 &= \sum_{s=1}^{126} \ln \{P(X_s = x_s | m_s, \alpha, \beta)\} \\
 &= \sum_{s=1}^{126} \ln \left\{ \binom{m_s}{x_s} \frac{B(\alpha + x_s, \beta + m_s - x_s)}{B(\alpha, \beta)} \right\} \\
 &= \sum_{s=1}^{126} \ln \left\{ \frac{m_s!}{(m_s - x_s)! x_s!} \frac{\Gamma(\alpha + x_s) \Gamma(\beta + m_s - x_s)}{\Gamma(\alpha + \beta + m_s)} \bigg/ \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)} \right\}
 \end{aligned}$$

The maximum value of the log-likelihood function is

$LL = -200.5$, which occurs at $\hat{\alpha} = 0.439$ and $\hat{\beta} = 95.411$.

Estimating Model Parameters

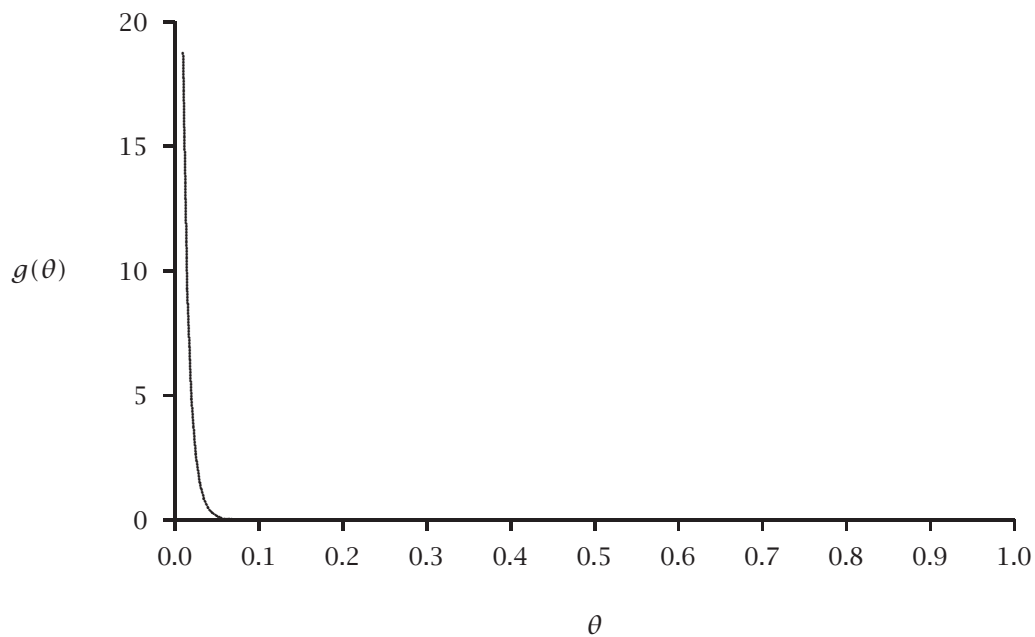
	A	B	C	D	E
1	alpha	1.000	B(alpha,beta)		1.000
2	beta	1.000			
3	LL	-718.9	← =SUM(E6:E131)		
4					
5	Segment	m_s	x_s	P(X=x m)	
6	1	34	0	0.02857	-3.555
7	2	102			
8	3	53			
9	4	145			
10	5	1254			
11					
12					
13					
14	9	1083	24	0.0009	-6.388
130	125	383	0	0.00260	-5.951
131	126	404	0	0.00247	-6.004

$$\begin{aligned}
 &= \text{EXP}(\text{GAMMALN}(B1) \\
 &\quad + \text{GAMMALN}(B2) \\
 &\quad - \text{GAMMALN}(B1+B2))
 \end{aligned}$$

$$\begin{aligned}
 &= \text{COMBIN}(B6,C6) * \text{EXP}(\text{GAMMALN}(B\$1 \\
 &\quad + C6) + \text{GAMMALN}(B\$2+B6-C6) - \\
 &\quad \text{GAMMALN}(B\$1+B\$2+B6)) / E\$1
 \end{aligned}$$

$$= \text{LN}(D11)$$

Estimated Distribution of Θ



$$\hat{\alpha} = 0.439, \hat{\beta} = 95.411, \widehat{E(\Theta)} = 0.0046$$

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Applying the Model

What is our best guess of θ_s given a response of x_s to a test mailing of size m_s ?

Intuitively, we would expect

$$E(\Theta_s | x_s, m_s) \approx \omega \frac{\alpha}{\alpha + \beta} + (1 - \omega) \frac{x_s}{m_s}$$

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Bayes' Theorem

- The *prior distribution* $g(\theta)$ captures the possible values θ can take on, prior to collecting any information about the specific individual.
- The *posterior distribution* $g(\theta|x)$ is the conditional distribution of θ , given the observed data x . It represents our updated opinion about the possible values θ can take on, now that we have some information x about the specific individual.
- According to Bayes' Theorem:

$$g(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\theta)g(\theta) d\theta}$$

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Bayes' Theorem

For the beta-binomial model, we have:

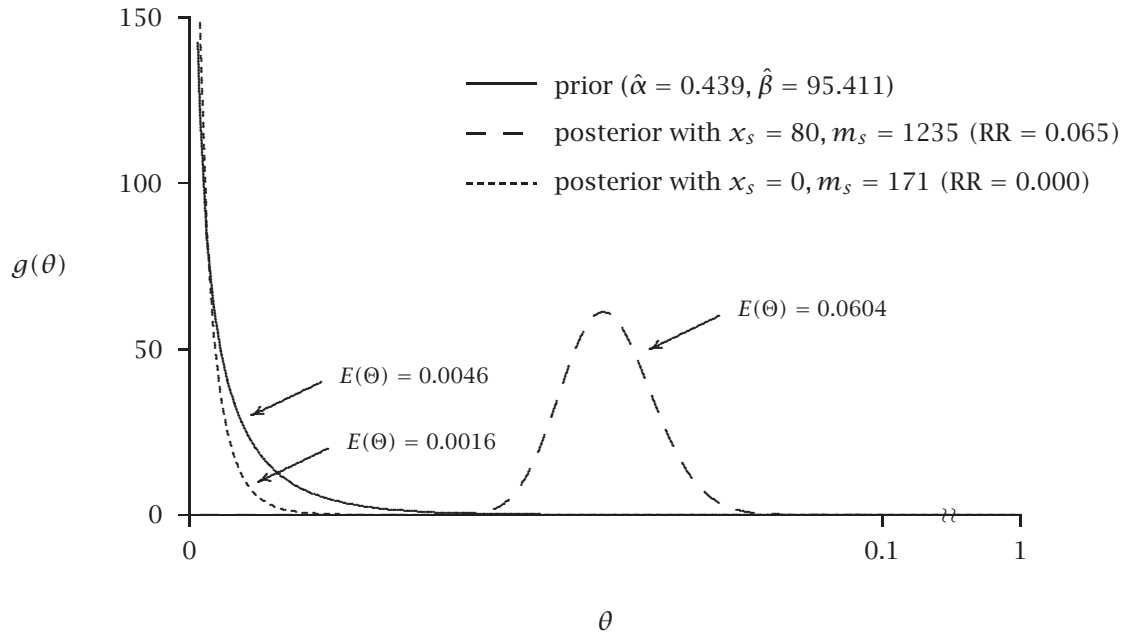
$$g(\theta_s|X_s = x_s, m_s) = \frac{\overbrace{P(X_s = x_s|m_s, \theta_s)}^{\text{binomial}} \overbrace{g(\theta_s)}^{\text{beta}}}{\underbrace{\int_0^1 P(X_s = x_s|m_s, \theta_s) g(\theta_s) d\theta_s}_{\text{beta-binomial}}}$$

$$= \frac{1}{B(\alpha + x_s, \beta + m_s - x_s)} \theta_s^{\alpha+x_s-1} (1 - \theta_s)^{\beta+m_s-x_s-1}$$

which is a beta distribution with parameters $\alpha + x_s$ and $\beta + m_s - x_s$.

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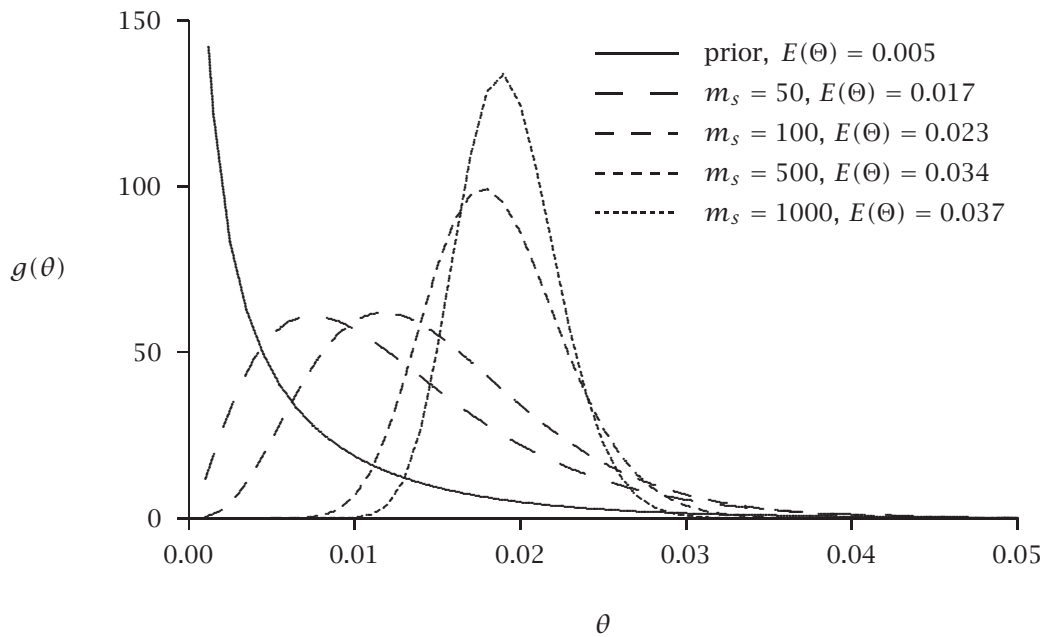
Distribution of Θ



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Impact of Sample Size on the Posterior

Four segments, each with a response rate of 0.04:



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Applying the Model

Recall that the mean of the beta distribution is $\alpha/(\alpha + \beta)$.
Therefore

$$E(\Theta_s | X_s = x_s, m_s) = \frac{\alpha + x_s}{\alpha + \beta + m_s}$$

which can be written as

$$\left(\frac{\alpha + \beta}{\alpha + \beta + m_s} \right) \frac{\alpha}{\alpha + \beta} + \left(\frac{m_s}{\alpha + \beta + m_s} \right) \frac{x_s}{m_s}$$

- a weighted average of the test RR (x_s/m_s) and the population mean ($\alpha/(\alpha + \beta)$).
- “Regressing the test RR to the mean”

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Model-Based Decision Rule

- Rollout to segments with:

$$E(\Theta_s | X_s = x_s, m_s) > \frac{3,343/10,000}{161.5} = 0.00207$$

- 66 segments pass this hurdle
- To test this model, we compare model predictions with managers’ actions. (We also examine the performance of the “standard” approach.)

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Results

	Standard	Manager	Model
# Segments (Rule)	51		66
# Segments (Act.)	46	71	53
Contacts	682,392	858,728	732,675
Responses	4,463	4,804	4,582
Profit	\$492,651	\$488,773	\$495,060

Use of model results in a profit increase of \$6,287;
126,053 fewer contacts, saved for another offering.

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Empirical Bayes Methods

- Bayesian analysis methods see us fixing the prior distribution before any data are observed.
- Empirical Bayes methods see us estimating the prior distribution from the data.
- When this prior has a parametric form, we are using parametric empirical Bayes methods.

“There is no one less Bayesian than an empirical Bayesian.”

Dennis Lindley

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Conjugate Priors

- When the posterior distribution comes from the same family as the prior distribution, the prior and posterior are called *conjugate distributions* and the prior is called the *conjugate prior* (\Rightarrow a closed-form expression for the posterior, which is mathematically convenient.)
- A distribution is a conjugate prior when its kernel is the same as that of the likelihood:

$$\begin{array}{cc} \text{prior} & \text{likelihood} \\ \hline \theta^{\alpha-1} (1 - \theta)^{\beta-1} & \theta^x (1 - \theta)^{n-x} \end{array}$$

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Concepts and Tools Introduced

- “Choice” processes
- The Beta Binomial model
- “Regression-to-the-mean” and the use of models to capture such an effect
- Bayes’ theorem and conjugate priors.
- The notion of (parametric) empirical Bayes methods.
- Using empirical Bayes methods in the development of targeted marketing campaigns

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Further Reading

Colombo, Richard and Donald G. Morrison (1988), "Blacklisting Social Science Departments with Poor Ph.D. Submission Rates," *Management Science*, **34** (June), 696-706.

Morwitz, Vicki G. and David C. Schmittlein (1998), "Testing New Direct Marketing Offerings: The Interplay of Management Judgment and Statistical Models," *Management Science*, **44** (May), 610-628.

Maritz, J.S. and T. Lwin (1989), *Empirical Bayes Methods*, 2nd edn, London: Chapman and Hall.

Discussion

Recap

The preceding four problems introduce simple models for three behavioral processes:

- Timing → “when / how long”
- Counting → “how many”
- “Choice” → “whether / which”

Phenomenon	Individual-level	Heterogeneity	Model
Timing (continuous)	exponential	gamma	Pareto Type II
Timing (discrete) (or counting)	geometric	beta	BG
Counting	Poisson	gamma	NBD
Choice	binomial	beta	BB

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Further Applications: Timing Models

- Response times:
 - Coupon redemptions
 - Survey response
 - Direct mail (response, returns, repeat sales)
- Other durations:
 - Salesforce job tenure
 - Length of web site browsing session
- Other positive “continuous” quantities (e.g., spend)

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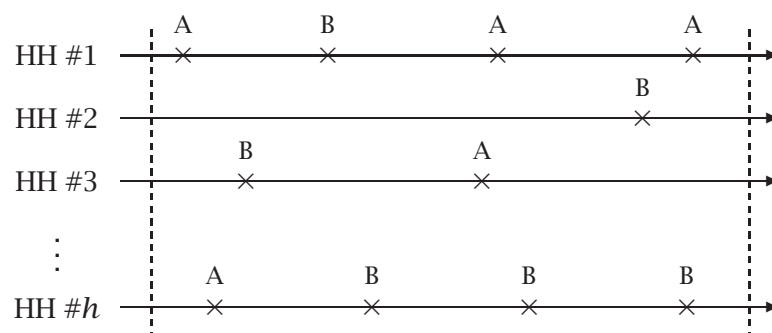
Further Applications: Count Models

- Exposure distributions (e.g., banner ads)
- Number of page views during a web site browsing session

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Further Applications: “Choice” Models

- Brand choice



- Multibrand choice (BB \rightarrow Dirichlet Multinomial)
- Media exposure
- Taste tests (discrimination tests)
- “Click-through” behavior

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Integrated Models

More complex behavioral phenomena can be captured by combining models from each of these processes:

- Counting + Timing
 - catalog purchases (purchasing| “alive” & “death” process)
 - “stickiness” (# visits & duration/visit)
- Counting + Counting
 - purchase volume (# transactions & units/transaction)
 - page views/month (# visits & pages/visit)
- Counting + Choice
 - brand purchasing (category purchasing & brand choice)
 - “conversion” behavior (# visits & buy/not-buy)

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A Template for Integrated Models

		Stage 2		
		Counting	Timing	Choice
Stage 1	Counting			
	Timing			
	Choice			

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Further Issues

- Relaxing usual assumptions:
 - – Non-exponential purchasing (greater regularity)
→ non-Poisson counts
 - – Non-gamma/beta heterogeneity (e.g., “hard core” nonbuyers, “hard core” loyals)
 - – Nonstationarity — latent traits vary over time
- The basic models are quite robust to these departures.

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Extensions

- Latent class/finite mixture models
- Introducing covariate effects
- Hierarchical Bayes (HB) methods

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The Excel spreadsheets associated with this tutorial, along with electronic copies of the tutorial materials, can be found at:

<http://brucehardie.com/talks.html>

An annotated list of key books for those interested in applied probability modelling can be found at:

<http://brucehardie.com/notes/001/>